Towards an operad of goals

Steve Huntsman

BAE Systems FAST Labs (goo.gl/Vrsqgm)

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Disclaimer

This is deliberately preliminary and hopefully provocative

Autonomous systems of systems (SoSs)

- Autonomous SoSs require us to design properties that are
  - Global ($\Rightarrow$ topology)
  - Emergent ($\Rightarrow$ statistical physics)
  - Compositional ($\Rightarrow$ category theory)
- Very rare to see work that applies even two of these fields
- Goal: sketch one way that all three might usefully contribute
  - Simplicial sets (roles), sheaves (data fusion), . . . skipping this
  - Renormalization
  - Operads
- Ideal: model SoSs with a compositional framework capable of capturing emergent and global properties
  - Account for variation through scoped parameters/settings

Statistical physics: renormalization "group"

- Physical perspective informed by Kadanoff, Wilson, *et al.*
  - Theory given by a function $f(\xi;\pi)$ of data ξ and parameters π
  - If for a suitable coarsening operator C there exists g s.t. $f(\xi;\pi) = f(C(\xi);g(\pi))$ then theory is renormalizable
  - Fixed point for infinite domains is of great physical interest
- The nLab features a very fancy QFT-oriented treatment that goes in the direction of refinement vs. coarsening
  - Higher-order Feynman diagrams (insertion operad!)
- We are not going to do *anything* fancy in what's next
  - f is a weighted average of finitely many function translates
  - ξ is the set of translation points; π is the weighting
  - C clusters translation points (BTW, single linkage is a functor)
  - g renormalizes a probability distribution based on clustering

The monopole operad

- Intuition: in far field, many point masses act like a single one
- Ingredients
  - Little disks operad
  - Probability operad
  - "Homogeneous" potential $V(r) := r^{-k}$ or $-\log r$
- Sources and weights $(\xi, \pi) \in (\mathbb{R}^d)^n \times \Delta^n$
- Operations: $V_{\xi,\pi}(x) := \sum_j \pi_j V(|x - \xi_j|) \sim V(|x|)$
- Composition also $\sim V(|x|)$:

$$V_{\xi,\pi} \circ \left(V_{\xi(1),\pi(1)}, \ldots, V_{\xi(n),\pi(n)} \right)(x) := \sum_j \pi_j V_{\xi(j),\pi(j)}(x - \xi_j)$$

So what?

- Ideal: model SoSs with a compositional framework suitable for engineering emergent and global properties
    - Account for details with scoped parameters, decorations, etc.
- Example: want a multiagent system that features
    - Distributed planning with $\ll n^2$ communications overhead
    - Robustness under addition/subtraction of agents
- Possible ingredients for (de)composing:
    - Tasks (flow graph operad)
        - Slides at math.ucr.edu/home/baez/ACT2017
        - Some notion of geometry necessary for motion planning
    - Constraints ("SAT operad")
    - Teams (simplicial operad)
        - Topology characterizes horizontal relationships
    - Communications (want an operadic framework!)

Artificial potentials

- Classic alternative to discretizing space for motion planning
    - Involves attractive (goal) and repulsive (obstacle) terms
        - Agents repel each other
        - Troublesome metastable minima can occur
        - Alas, the monopole operad can't handle signed terms
    - Total potential must represent complex spatial relationships
        - Many terms = expensive
    - Might still be useful for applications where space is discrete
        - E.g., graph Laplacian stuff
        - Not pulling on this thread here

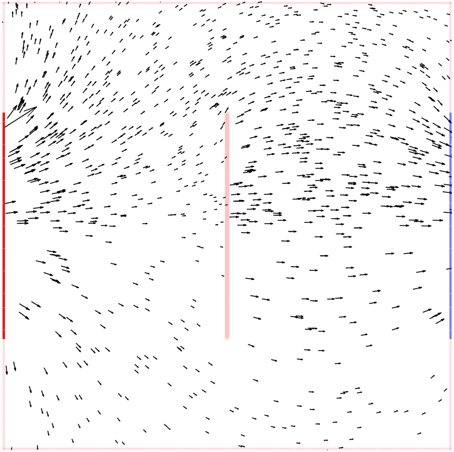
The fast multipole method (FMM)

- FMM [Greengard & Rokhlin 1987] accurately simulates N interacting particles in $O(N)$ (vs $O(N^2)$) time
  - Named one of the top 10 algorithms of the 20th century [Dongarra & Sullivan (2000)]
- Two key ideas (approximate renormalization!)
  - Represent clusters of particles with multipole expansions that are as coarse (i.e., truncated) as possible in far-field
    - For approximation error ε, truncate at $\lceil \log_2(1/\varepsilon) \rceil$ terms
    - Nearby particles interact directly
  - Decompose space hierarchically to get well-separated clusters
- Works for (at least) homogeneous and logarithmic potentials
- Parallel version [Greengard & Gropp 1990] exploits locality

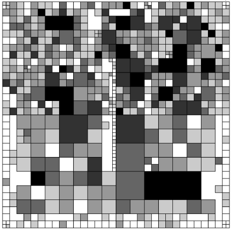
The fast multipole method (FMM)

Eval points $\sim \frac{4}{5}U(\text{top half}) + \frac{1}{5}U(\text{bottom half})$

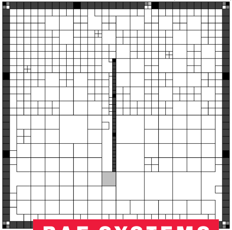
Red: + source; blue: - source; black: gradient evals



Relative # of eval points per leaf box (black = max)



Relative # of source points per leaf box (black = max)

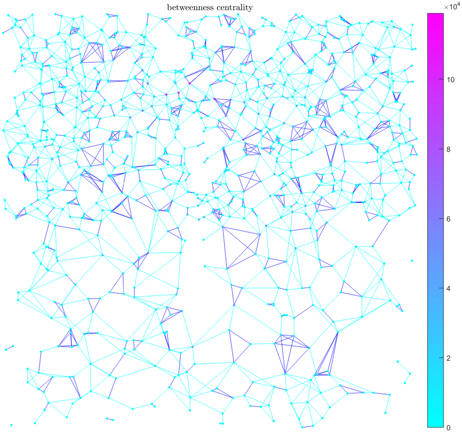


A multipole operad?

- The FMM builds a tree whose nodes are decorated with truncated multipoles
  - Approximates finer quadtree with monopoles on leaves
  - Tree topology doesn't depend on charge values
- We'll show how to use the FMM to build networks that seem to play nicely with
  - Little cubes operad
  - Motion plans
- This points towards our desiderata, viz.
  - Distributed planning with $\ll n^2$ communications overhead
  - Robustness under addition/subtraction of agents
- Exercise: how much of this can be realized operadically?

Coordination topology

- Connect all nodes in the same leaf box
- Connect nearest nodes in adjacent boxes
- Connect orphans to nearest nodes (not shown)



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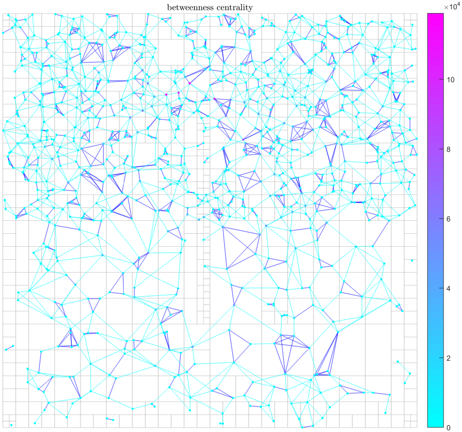
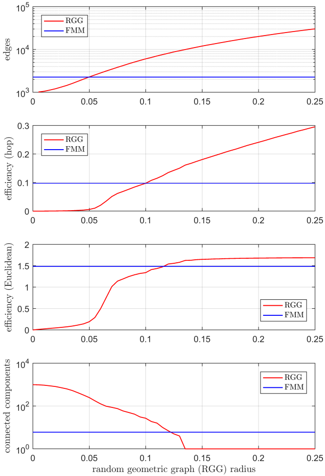
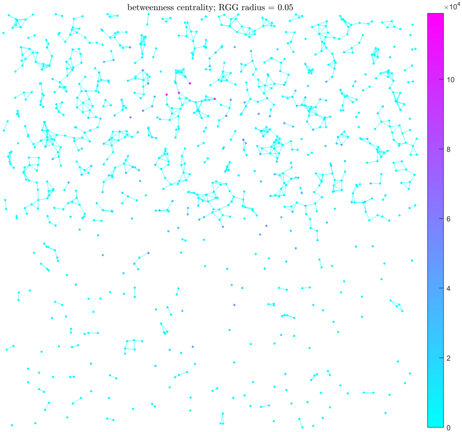


Comparison with random geometric graphs





$$\text{efficiency} = \langle d_{jk}^{-1} \rangle_{j \neq k}$$

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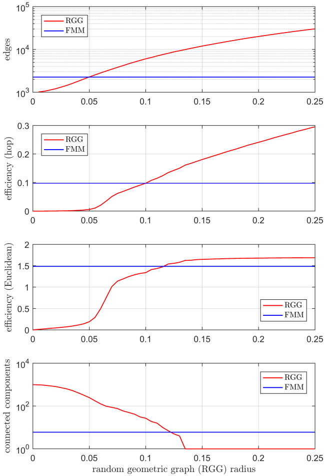

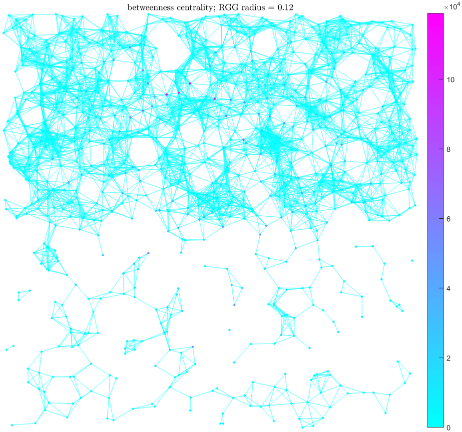

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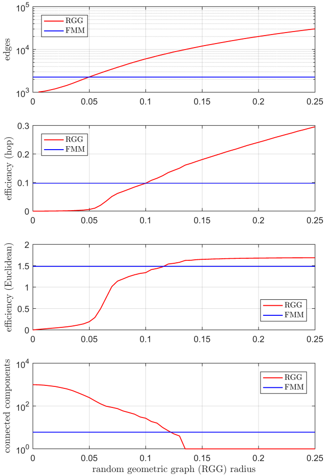

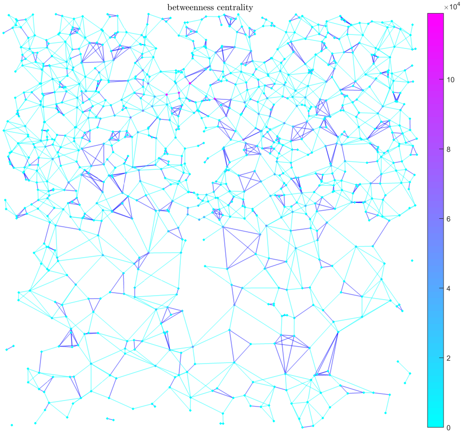

$$\text{efficiency} = \langle d_{jk}^{-1} \rangle_{j \neq k}$$

Towards an operad of goals

- FMM-based framework for networks and motion plans
  - How much of this can be realized operadically?
- Flow graph operad for tasks
- SAT operad for constraints
  - Weighted MAX-SAT for optimization
- Simplicial operad for teams and requirements
  - Matching a team to requirements amounts to finding an optimal (not necessarily surjective) simplicial map
- Topology, statistical physics, and category theory can (and likely will) have something to contribute to the development of autonomous systems of systems

Thanks

BAE Systems FAST Labs (goo.gl/Vrsqgm)

steve.huntsman@baesystems.com

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Moving parts for classical case

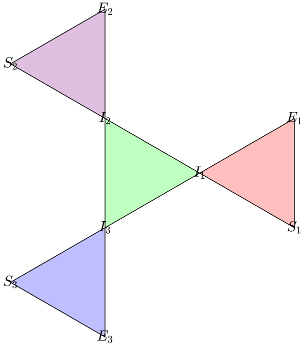
- dim $= 2$, $V = -\log$, charges q_j at $z_j \in \mathbb{C}$; take real parts
- Set $Q := \sum_j q_j$; $A := \sum_j |q_j|$; $a_\ell := -\sum_j q_j z_j^\ell / \ell$; $r > \max |z_j|$
- Multipole expansion with error bounds:

$$\left| \sum_j q_j \log(z - z_j) - Q \log z - \sum_{\ell=1}^{L} \frac{a_\ell}{z^\ell} \right| \overset{|z|>r}{\leq} \frac{A}{\left(\frac{|z|}{r} - 1\right)\left(\frac{|z|}{r}\right)^L}$$

- More involved error bounds for shifting centers of multipole expansions, converting them to Taylor series, and shifting the Taylor expansions

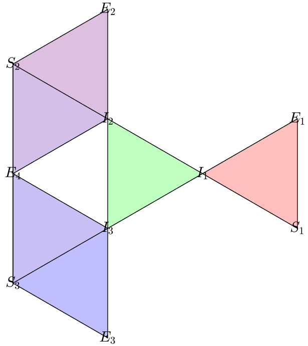
Topology: simplicial sets, sheaves, etc.

Subsystem interfaces I_j
Sensors S_j
Effectors E_j



Topology: simplicial sets, sheaves, etc.

k-simplices: roles w/ $k-1$ players
Subsystem interfaces I_j
Sensors S_j
Effectors E_j
Fusing sensors $\Rightarrow$ nontrivial homology



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