A short introduction to a general theory of interactivity

Stéphane Dugowson
Supmeca (Paris)

Applied Category Theory Workshop
NIST / Gaithersburg, MD
March 15 & 16, 2018

A short introduction to a general theory of interactivity

taking the Conway's Game of Life as a guiding thread

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Global models

GoL Rules

GoL Rules

- 3 neighbors → 1 (= alive)
- 2 neighbors → unchanged state
- otherwise → 0 (= dead)

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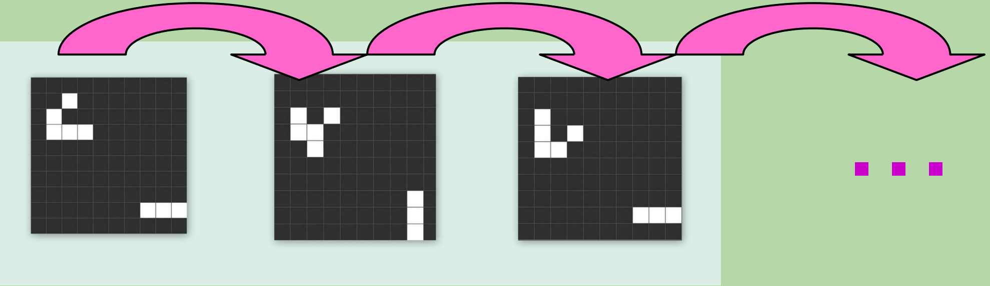
- 2 neighbors → unchanged state
- otherwise → 0 (= dead)

Example



Global models

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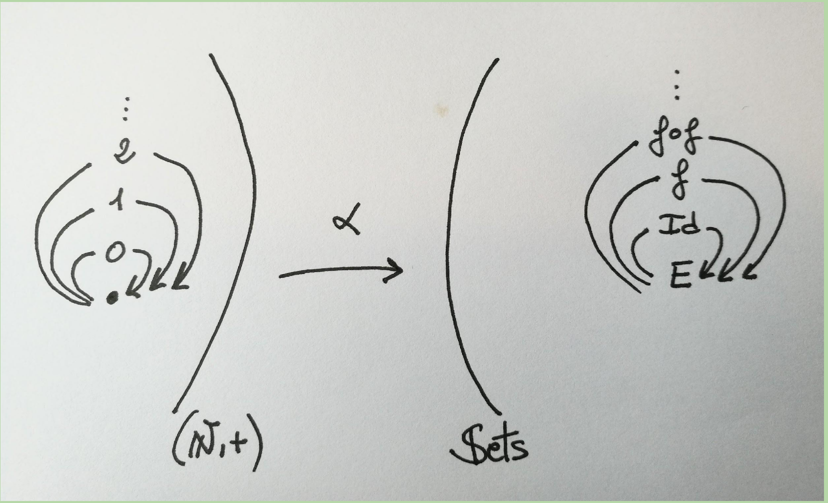
A discrete dynamical system

The Conway's game of life as a functor



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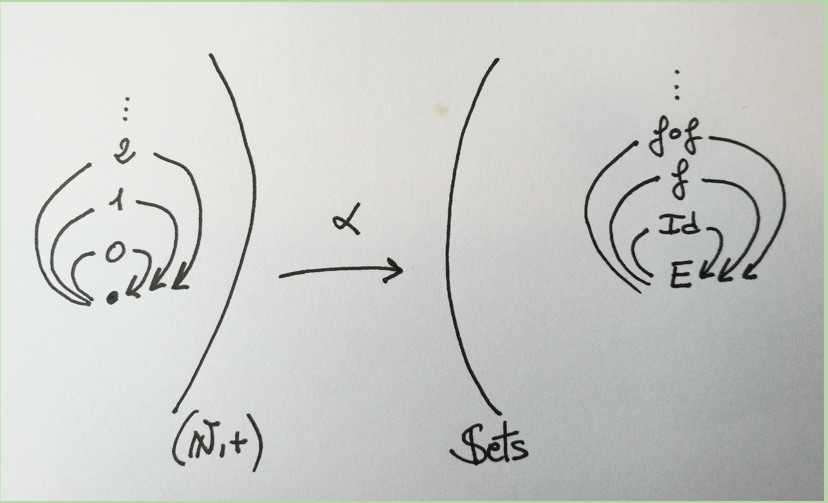

with $E = \{0,1\}^{(\mathbb{Z}^2)}$ and $\left|\begin{array}{l} f : E \longrightarrow E \\ (x_{ij})_{\mathbb{Z}^2} \longmapsto (y_{ij})_{\mathbb{Z}^2} \end{array}\right.$

with $\forall ij$ $\begin{cases} \tau_{ij} = 3 \Rightarrow y_{ij} = 1 \\ \tau_{ij} = 2 \Rightarrow y_{ij} = x_{ij} \\ \text{otherwise, } y_{ij} = 0 \end{cases}$ where $\tau_{ij} = \sum_{k,l \in \mathcal{N}_{ij}} x_{kl}$

and $\mathcal{N}_{ij} = \left\{ k,l \neq ij, \begin{array}{l} |k-i| \leq 1 \\ |l-j| \leq 1 \end{array} \right\}$

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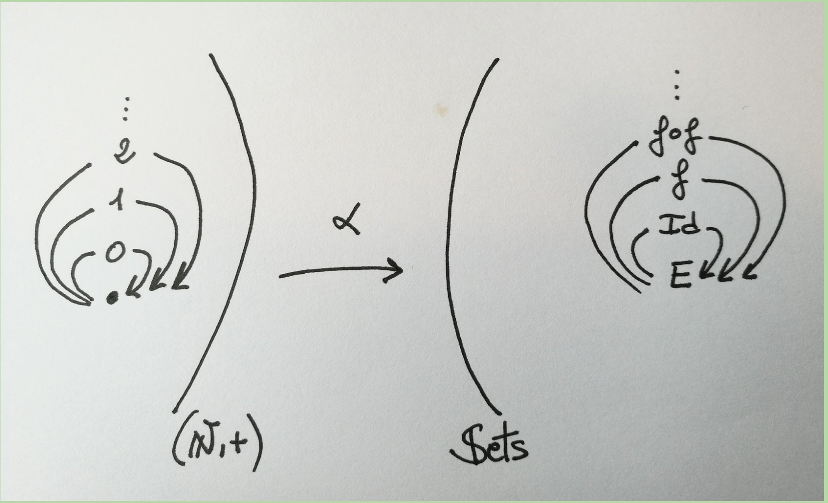
The Conway's game of life as a functor



(Remark : discrete dynamical systems constitute a topos of presheaves

$$\widehat{(\mathbb{N},+)} = \mathbf{Sets}^{(\mathbb{N},+)}$$)

The one-step global model

$$\left(\underset{0}{\cdot} \xrightarrow{d} \underset{1}{\cdot} \right)_{\mathbb{D}} \xrightarrow{\alpha} \left(E_0 \xrightarrow{d^{\alpha}} E_1 \right)$$

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The one-step global model

$$\left(\underset{0}{\cdot} \xrightarrow{d} \underset{1}{\cdot}\right)_{\mathbb{D}} \xrightarrow{\alpha} \left(E_0 \xrightarrow{d^\alpha} E_1\right)$$

with $E_k = \{0_k, 1_k\}^{\mathbb{Z}^2}$

where

$$\begin{cases} 0_k = (0,k) \\ 1_k = (1,k) \\ \text{and } [0_k] = 0 \\ \qquad [1_k] = 1 \end{cases}$$

and $d^\alpha : E_0 \longrightarrow E_1$ defined by

$$(x_{ij})_{\mathbb{Z}^2} \longmapsto (y_{ij})_{\mathbb{Z}^2}$$

$$\forall ij \quad \begin{cases} \tau_{ij} = 3 \Rightarrow y_{ij} = 1_1 \\ \tau_{ij} = 2 \Rightarrow [y_{ij}] = [x_{ij}] \\ \text{otherwise, } \quad y_{ij} = 0_1 \end{cases}$$

where $\tau_{ij} = \sum_{(k,l) \in \mathcal{N}_{ij}} [x_{kl}]$

and $\mathcal{N}_{ij} = \left\{ (k,l) \neq (i,j), \begin{array}{l} |k-i| \leq 1 \\ |j-l| \leq 1 \end{array} \right\}$

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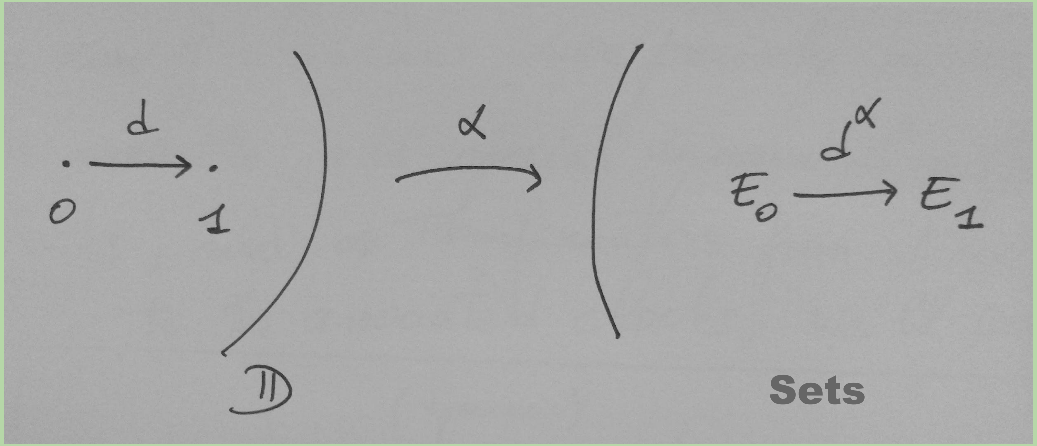
Functorial dynamics

Functorial dynamics



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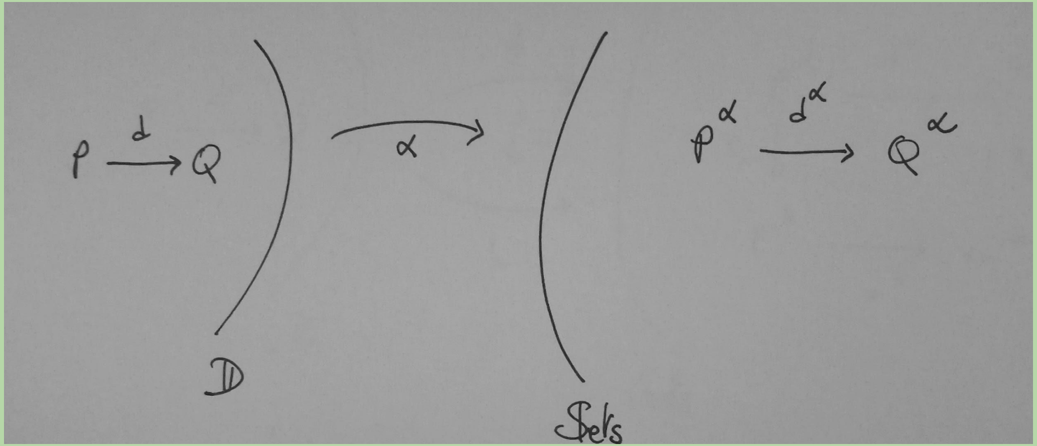

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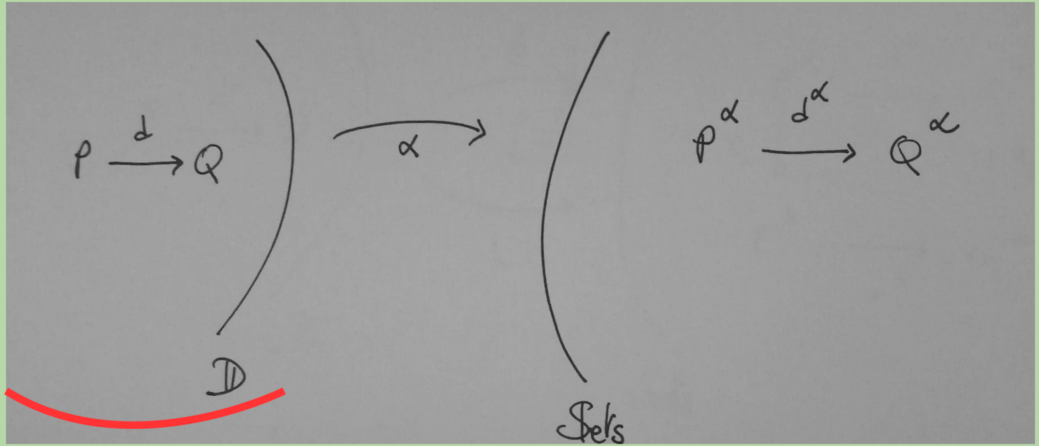
engine

Functorial dynamics



The *engine* of α

Global models

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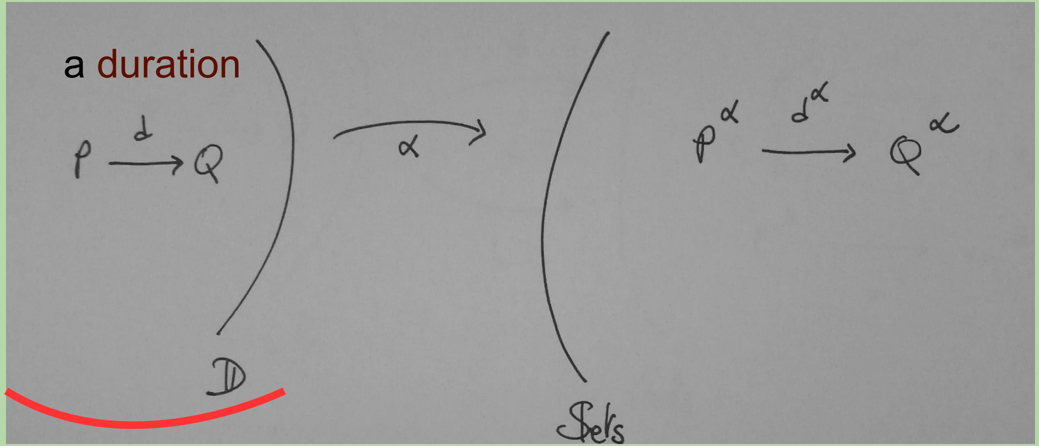
Functorial dynamics

engine, durations,

Functorial dynamics



The *engine* of α

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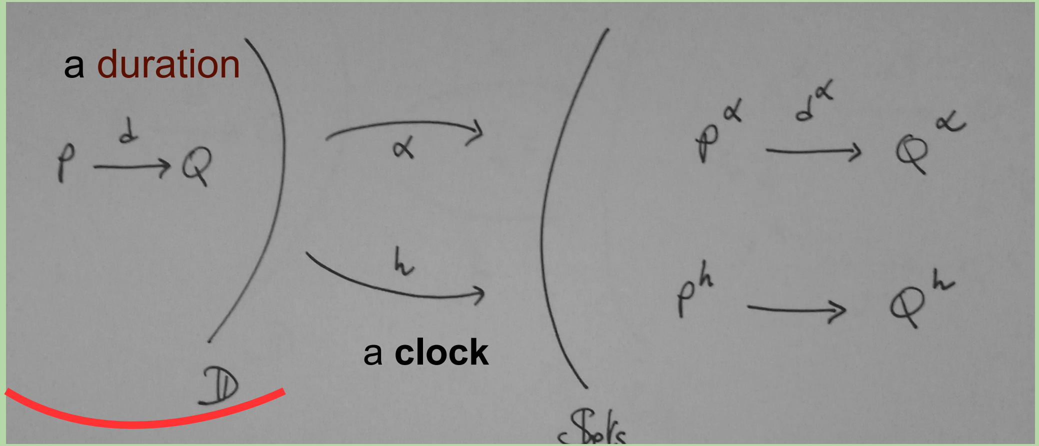
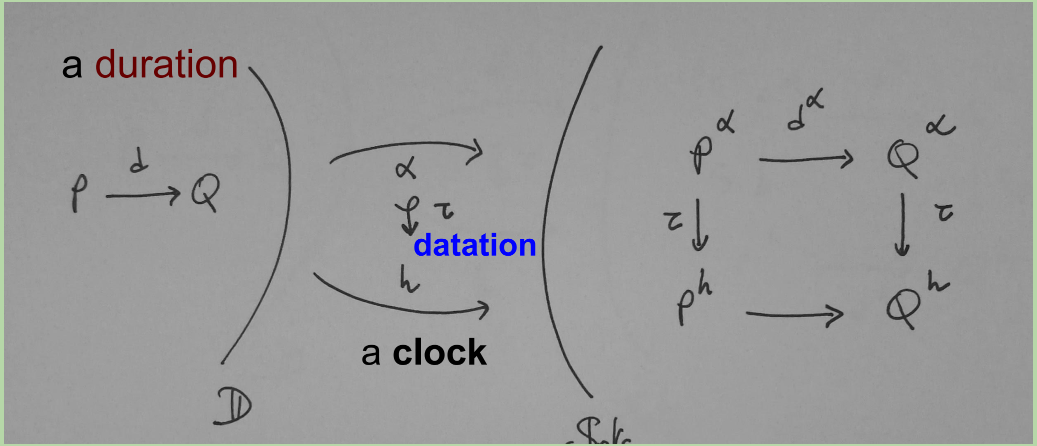
engine, durations, clock,



The *engine* of α

Functorial dynamics



The *engine* of α

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engine, durations, clock, datation,

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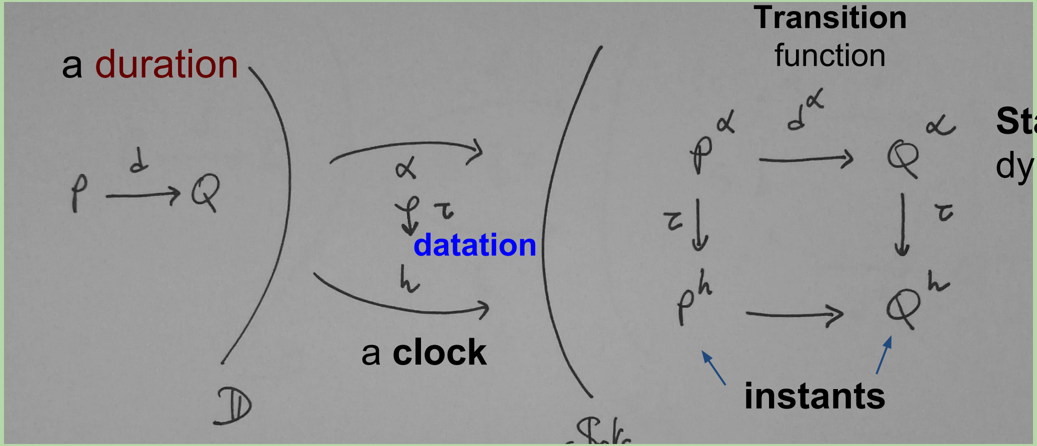
engine, durations, clock, datation, instants,

Functorial dynamics



States of the dynamic

The *engine* of α

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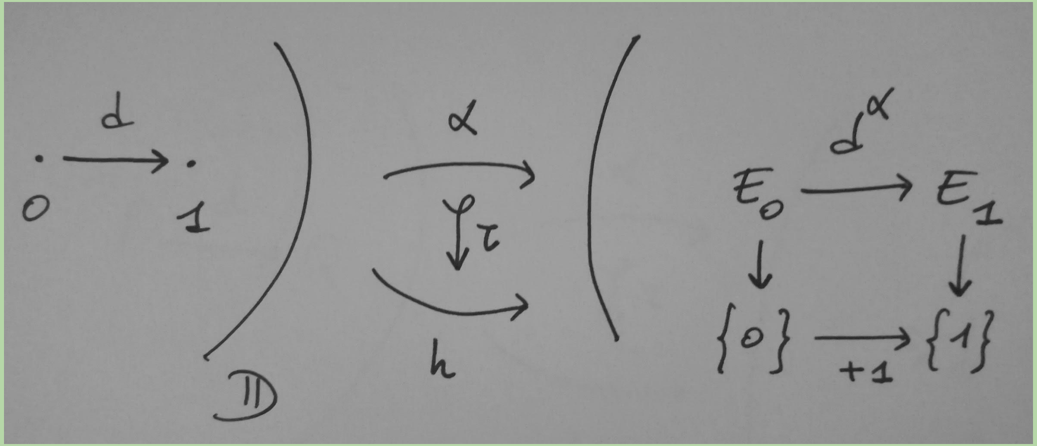
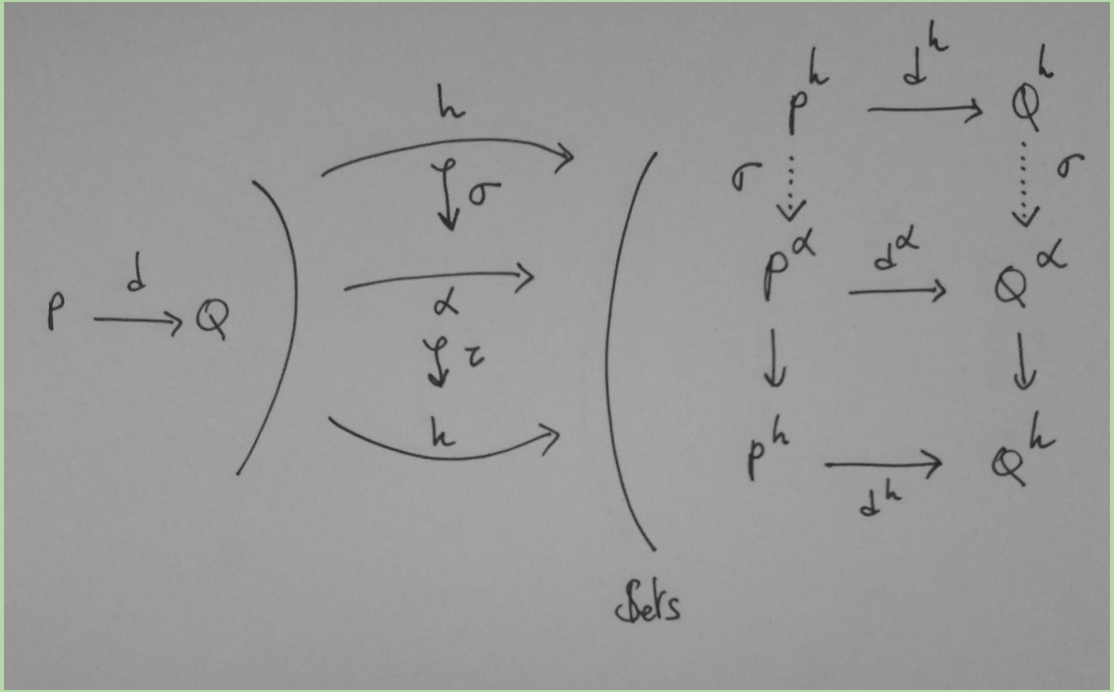
Functorial dynamics

engine, durations, clock, datation, instants,

Functorial dynamics



Functorial dynamics : realizations



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Functorial dynamics

engine, durations, clock, datation, Instants, realizations

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engine, durations, clock, datation, Instants, realizations

Functorial dynamics : realizations

$$\begin{tikzcd}[column sep=huge]
(P \xrightarrow{d} Q) \arrow[r, bend left=50, "h"{name=U}] \arrow[r, "\alpha"{name=M, description}] \arrow[r, bend right=50, "h"'{name=D}] & \mathbf{Sets}
\arrow[from=U, to=M, Rightarrow, "\sigma"] \arrow[from=M, to=D, Rightarrow, "\tau"]
\end{tikzcd}
\qquad
\begin{tikzcd}
P^h \arrow[r, "d^h"] \arrow[d, dashed, "\sigma"'] & Q^h \arrow[d, dashed, "\sigma"] \\
P^\alpha \arrow[r, "d^\alpha"] \arrow[d] & Q^\alpha \arrow[d] \\
P^h \arrow[r, "d^h"'] & Q^h
\end{tikzcd}$$

$$\tau \circ \sigma \subseteq Id$$

that is

$$\begin{cases} \tau_P \circ \sigma_P \subseteq Id_{P^h} \\ \tau_Q \circ \sigma_Q \subseteq Id_{Q^h} \end{cases}$$

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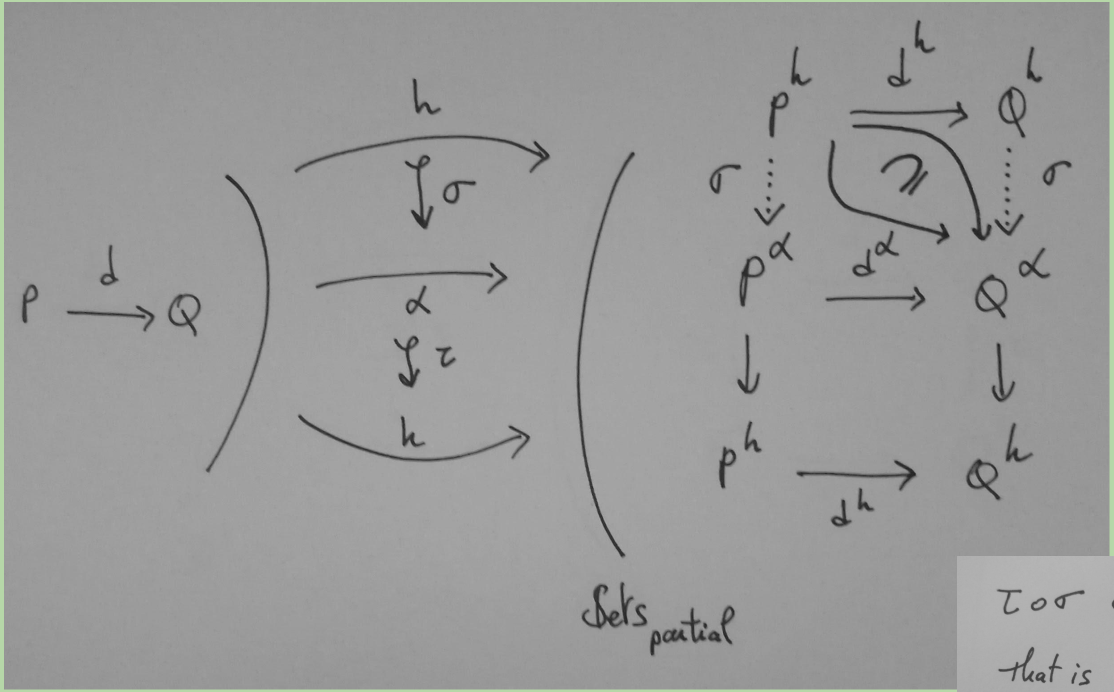
Functorial dynamics : realizations



$\tau \circ \sigma \subseteq Id$

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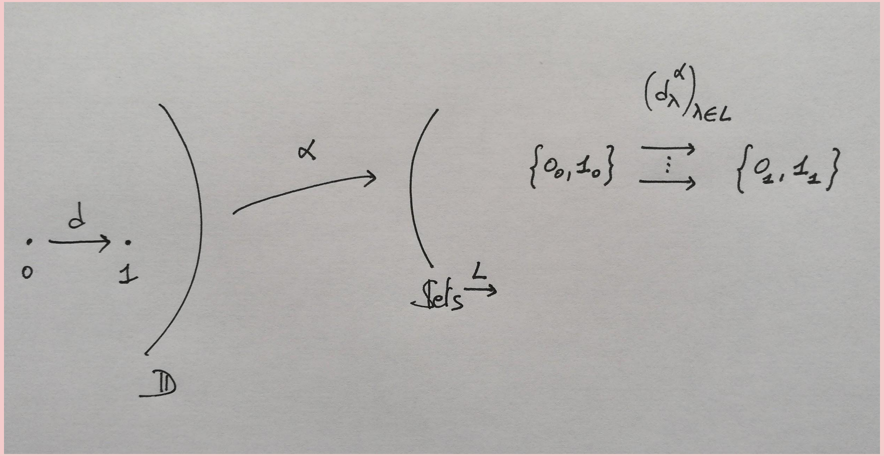
An individual cell of the (one-step) Game of Life

Components
(open dynamics)

---

parameters

a cell of the GoL



Components
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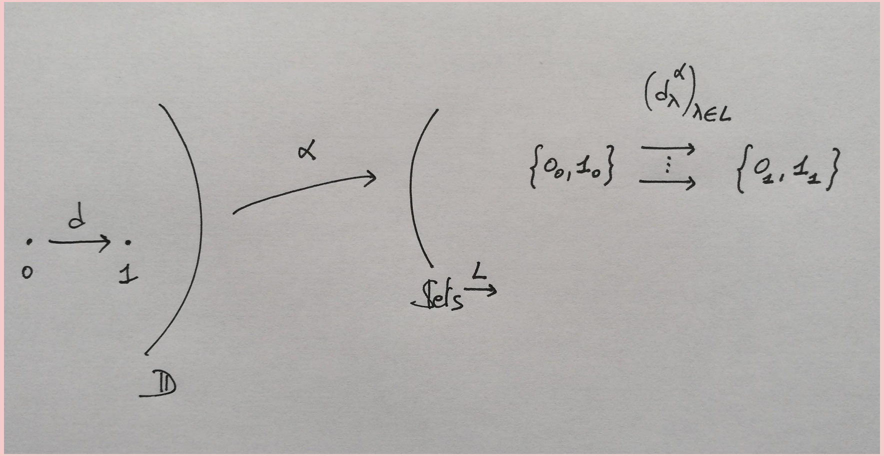
An individual cell of the (one-step) Game of Life



with . $L=\{0,1,2,\dots,8\}$,

. $(\mathbf{Sets}^{L\to})$ is the category with
- . <u>Sets</u> as objects
- . <u>L-families</u> of functions as arrows

. and : $d_3^\alpha \equiv 1_1$, $[d_2^\alpha(x)]=[x]$

and $d_\lambda^\alpha \equiv 0_1$ for others λ.

An individual cell of the (one-step) Game of Life

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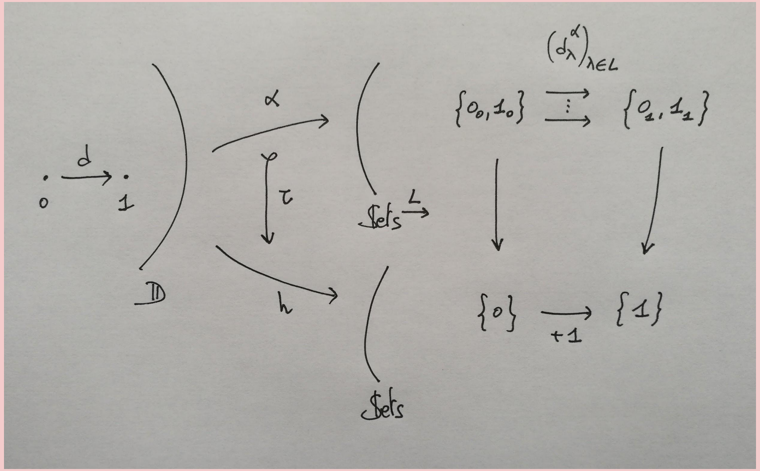

with . $L = \{0, 1, 2, \dots, 8\}$,

. $(\mathcal{S}ets^{\xrightarrow{L}})$ is the category with
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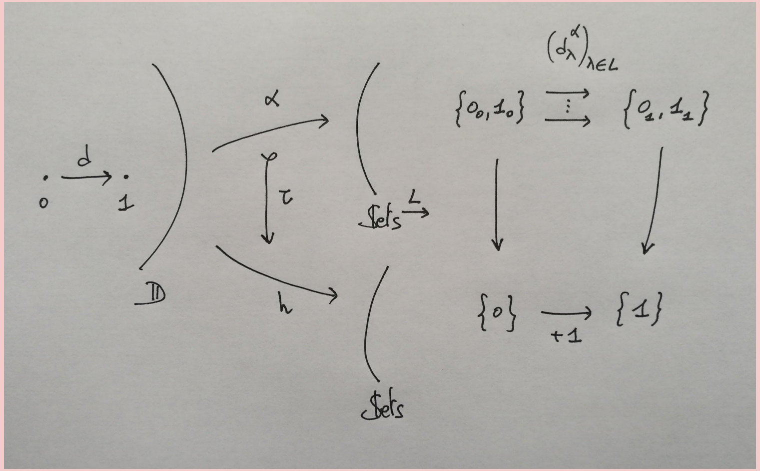
An indeterministic cell of the Game of Life ?

Components
(open dynamics)

parameters

a cell of the GoL

indeterminism



Components
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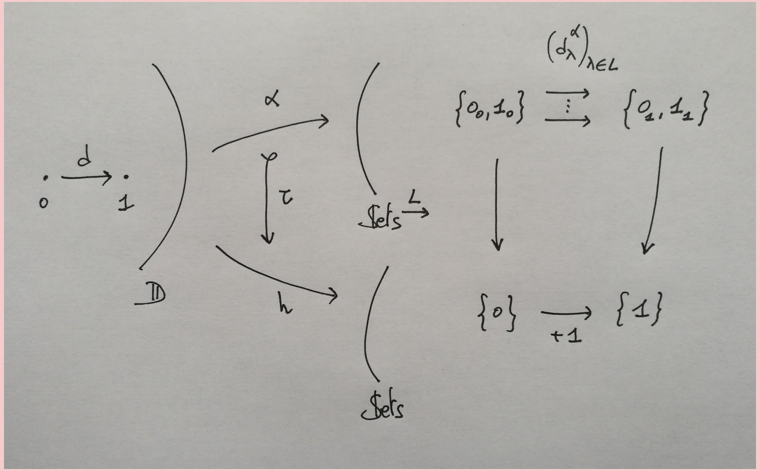
An indeterministic cell of the Game of Life ?



A non-determinist transition $A \overset{f}{\rightsquigarrow} B$ is a function $A \xrightarrow{f} \mathcal{P}(B)$, (or a binary relation $A \to B$).

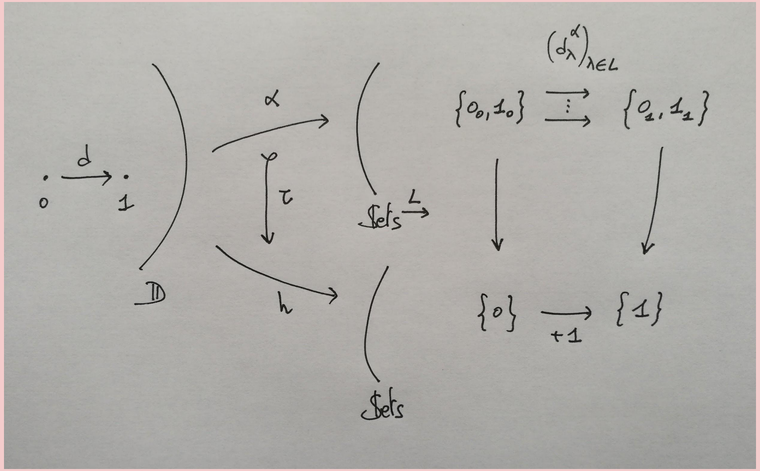
An indeterministic cell of the Game of Life ?

Components
(open dynamics)

parameters

a cell of the GoL

indeterminism



A non-determinist transition $A \overset{f}{\rightsquigarrow} B$
is a function $A \xrightarrow{f} \mathcal{P}(B)$,
(or a binary relation $A \longrightarrow B$).

$\mathbb{P} \simeq$ Rel = the category with
- sets
- non-determinist transitions = binary relations

An indeterministic cell of the Game of Life

Components
(open dynamics)

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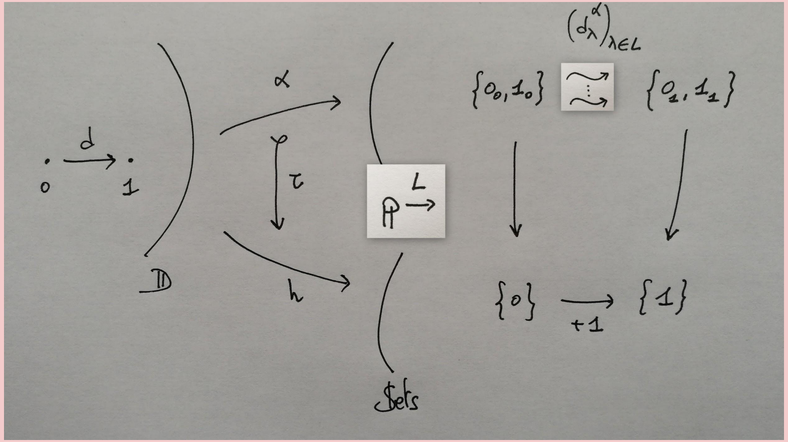

A non-deterministic transition $A \overset{f}{\rightsquigarrow} B$ is a function $A \xrightarrow{f} \mathcal{P}(B)$, (or a binary relation $A \to B$).

$\mathbb{P} \simeq \mathbf{Rel}$ = the category with
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- non-deterministic transitions = binary relations

Components
(open dynamics)

parameters

a cell of the GoL

indeterminism

definition of
an open
functorial dynamic

Definition of an open functorial dynamic

<u>An open (functorial) dynamic</u>

$$\left(\mathbb{D} \xrightarrow{\alpha} \mathbb{P}^{\xrightarrow{L}}\right) \xrightarrow{\tau} \left(\mathbb{P} \xrightarrow{h} \mathbf{Sets}\right)$$

Components
(open dynamics)

parameters

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indeterminism

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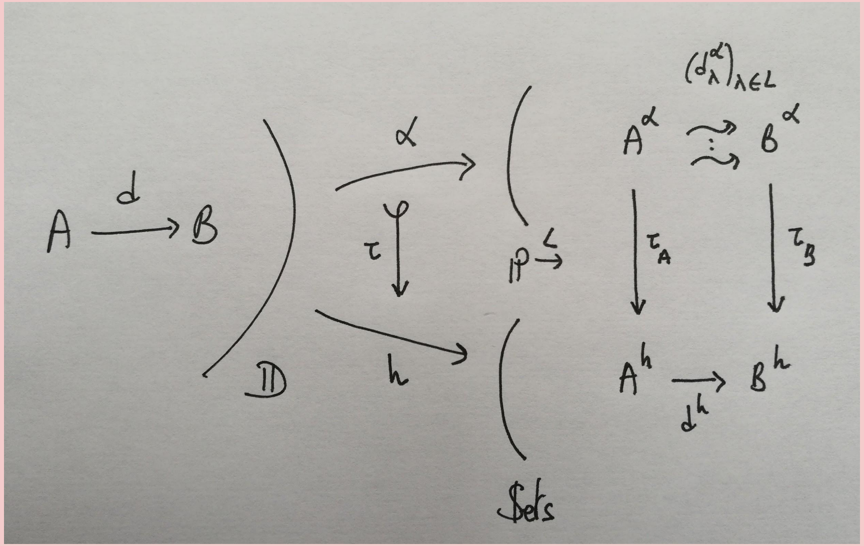
Definition of an open functorial dynamic



An open (functorial) dynamic

$$\left(\mathbb{D} \xrightarrow{\alpha} \mathbb{P}^{\mathcal{L}}\right) \xrightarrow{\tau} \left(\mathbb{D} \xrightarrow{h} \mathcal{S}ets\right)$$

Components
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definition of
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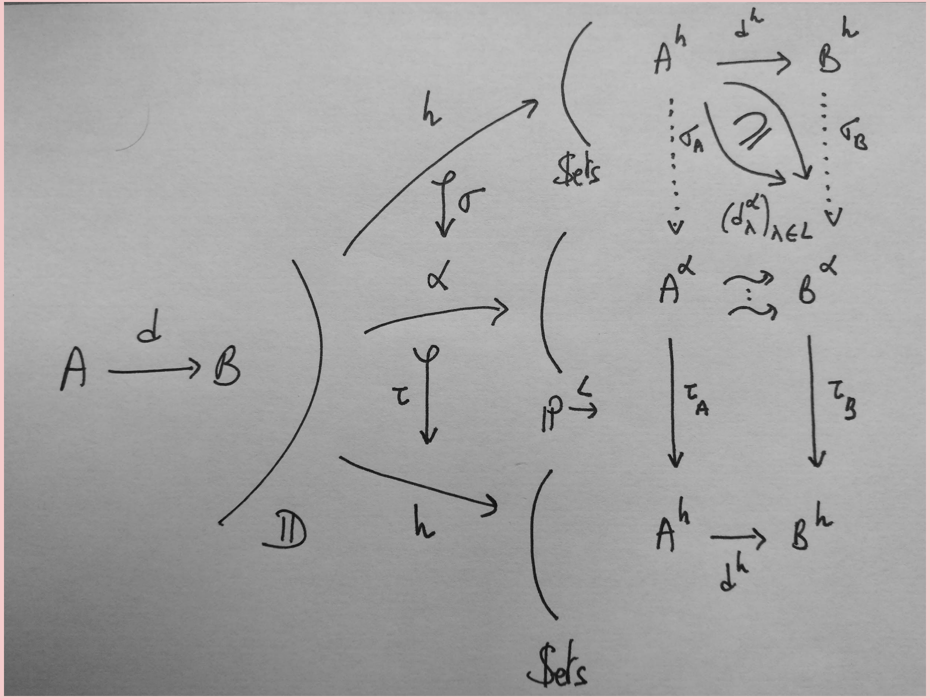
realizations

Realizations of an open dynamic



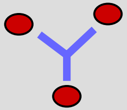
A realization of (α, τ)
is (λ, σ)
with $\lambda \in L$
and $\sigma : h \to \alpha$
s.t. $\tau \circ \sigma \subseteq Id_h$
and $\sigma_B \circ d^h \subseteq d^\alpha_\lambda \circ \sigma_A$

Interactive families

An *interactive family* $\mathcal{F}$ will be given by

- a family of open dynamics $(\mathcal{A}_i)_{i \in I}$

$$\left(\mathcal{A}_i = \left(\left(\mathbb{D}_i \xrightarrow{d_i} \mathbb{P} \xrightarrow{L_i} \right) \xrightarrow[\tau_i]{} \left(\mathbb{D}_i \xrightarrow{h_i} \mathbf{Sets} \right) \right) \right)_{i \in I}$$



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$$\left(\mathcal{A}_i = \left((\mathbb{D}_i \xrightarrow{d_i} \mathbb{P} \xrightarrow{L_i}) \xrightarrow[\tau_i]{} (\mathbb{D}_i \xrightarrow{h_i} \mathcal{S}ets)\right)\right)_{i\in I}$$

- a *synchronization*,

Interactive families

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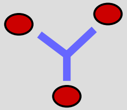
$$\left(\mathcal{A}_i = \left((\mathbb{D}_i \xrightarrow{d_i} \mathbb{P} \xrightarrow{L_i}) \xrightarrow[\tau_i]{\varphi} (\mathbb{D}_i \xrightarrow{h_i} \mathcal{S}ets) \right)\right)_{i \in I}$$

- a *synchronization*,
- an *interaction.*



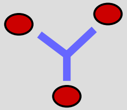
Interactive families

synchronization

Synchronization

A *synchronization* for a family $(\mathcal{A}_i)_{i \in I}$ of open dynamics is given by :

- An index $i_0 \in I$, called the *conductor*,
- A family of functors and monotonic maps which associate instants of clocks to the instants of the conductor



Interactive families

synchronization

interaction

Interaction

An *interaction* is defined as a multiple relation between realizations and parameters.

Interactive families

synchronization

interaction

Interaction

An *interaction* is defined as a multiple relation between realizations and parameters.

For example :

$$|R| = \left\{ \begin{pmatrix} \lambda_{ij} \\ \sigma_{ij} \end{pmatrix}_{(i,j)\in \mathbb{I}^2} \text{ such that } \begin{cases} \lambda_{ij} = \sum_{(k,l)\in \mathcal{N}_{ij}} \sigma_{kl}(0) \\ \sigma_{ij} \in \mathcal{S}_{\lambda_{ij}} \end{cases} \right\}$$

where $\mathcal{S}_{\lambda_{ij}}$ is the set of realizations of the open cell α for the parameter value λ_{ij}

Interactive families

synchronization

interaction

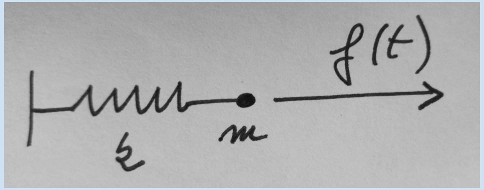
a remark
about future

Interaction

An *interaction* is defined as a multiple relation
between realizations and parameters.

Generally,
parameters are linked
not only with *“current” states*,
but with *whole realizations*
of the concerned dynamics.

Interaction



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synchronization

interaction

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Interactive families

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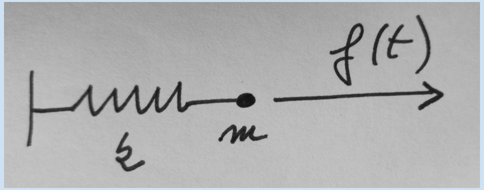
Interaction



$$\begin{cases} m x''(t) + k x(t) = f(t) \\ x(0) = x_0 \\ x'(0) = v_0 \end{cases}$$

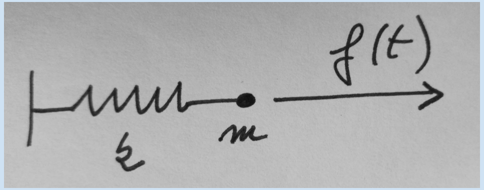
Interactive families

synchronization

interaction

a remark about future

Interaction



$$\begin{cases} m\,x''(t) + k\,x(t) = f(t) \\ x(0) = x_0 \\ x'(0) = v_0 \end{cases}$$

$$x(t) = \frac{1}{\sqrt{mk}} \int_{s=0}^{s=t} \sin\left(\sqrt{\frac{k}{m}}\,(t-s)\right) f(s)\,ds$$

$$+ \sqrt{\frac{m}{k}}\, v_0 \sin\left(\sqrt{\frac{k}{m}}\, t\right) + x_0 \cos\left(\sqrt{\frac{k}{m}}\, t\right)$$

Interactive families

synchronization

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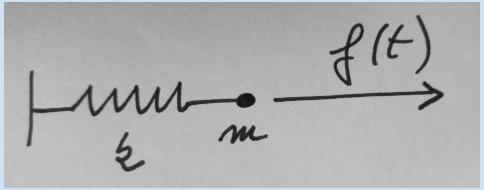
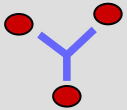
Interaction

$$\begin{cases} m x''(t) + k\, x(t) = f(t) \\ x(0) = x_0 \\ x'(0) = v_0 \end{cases}$$



$$x(t) = \frac{1}{\sqrt{m k}} \int_{s=0}^{\boxed{s=t}} \sin\left(\sqrt{\frac{k}{m}}\,(t-s)\right) \boxed{f(s)}\, ds + \sqrt{\frac{m}{k}}\, v_0 \sin\left(\sqrt{\frac{k}{m}}\, t\right) + x_0 \cos\left(\sqrt{\frac{k}{m}}\, t\right)$$



Interactive families

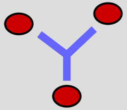
synchronization

interaction

a remark about future

Interaction

the future will often depend on the future



Interactive families

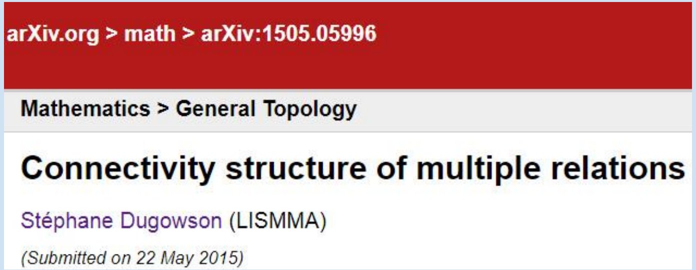
synchronization

interaction

a remark about future

connectivity

About the connectivity structure of interactions



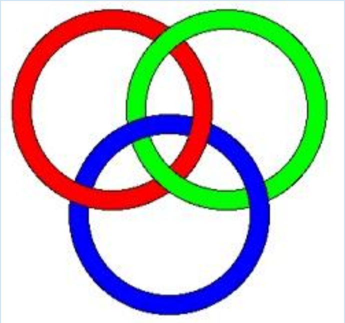
arXiv.org > math > arXiv:1505.05996

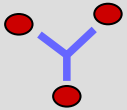
Mathematics > General Topology

Connectivity structure of multiple relations

Stéphane Dugowson (LISMMA)

(Submitted on 22 May 2015)





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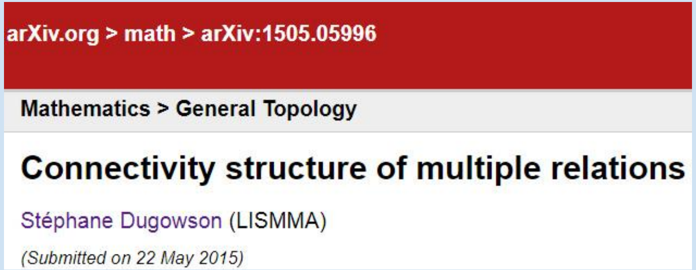
About the connectivity structure of interactions



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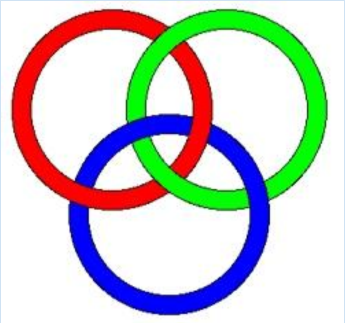
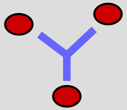
(Submitted on 22 May 2015)



Interaction → Connectivity structures



Interactive families

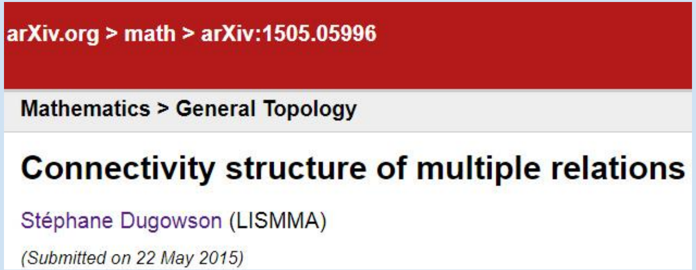
synchronization

interaction

a remark about future

connectivity

About the connectivity structure of interactions



arXiv.org > math > arXiv:1505.05996

Mathematics > General Topology

Connectivity structure of multiple relations

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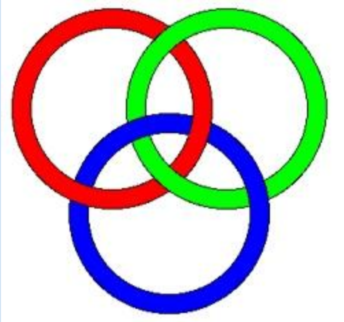

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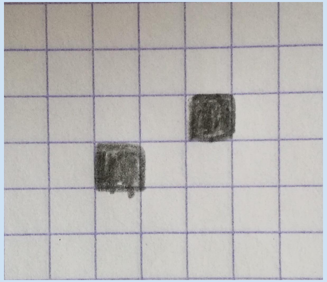

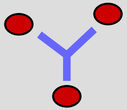

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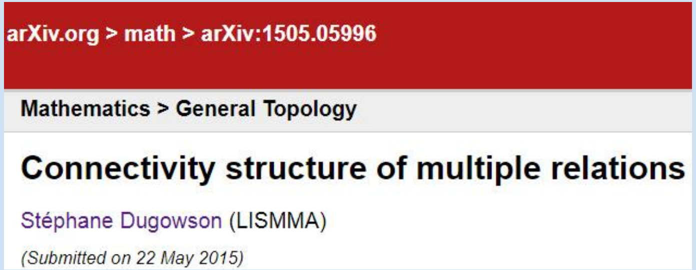

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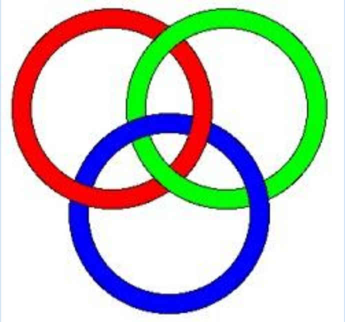
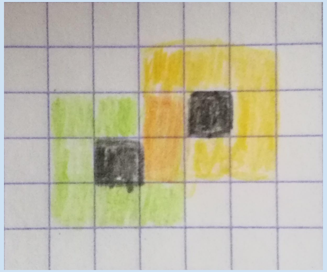


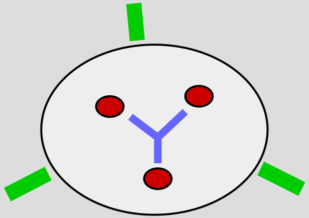
Interaction

Connectivity structures

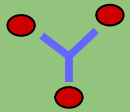


Interactive
families

Components
(open dynamics)



Generated global dynamics



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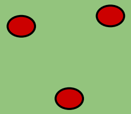

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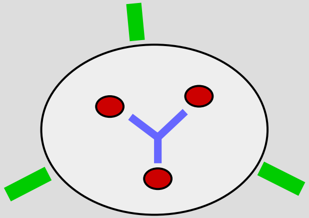

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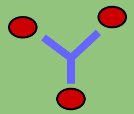

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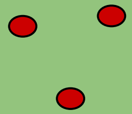
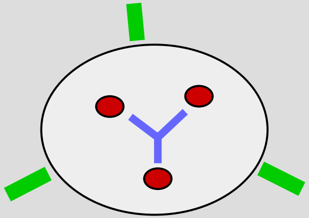


Components (open dynamics)

The generated dynamics are only “sub-functorial”.



Generated global dynamics

relax !

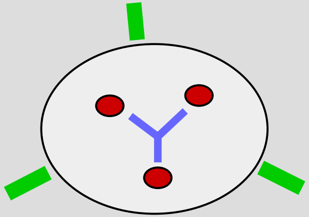
sub-functorial dynamics

Interactive families

Components (open dynamics)

The generated dynamics are only “sub-functorial”.

(this can be interpreted as a kind of lax-functoriality)



Generated global dynamics

---

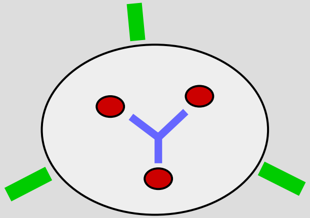
relax !

sub-functorial dynamics

Sub-functorial dynamics

$$\left(A \xrightarrow{d} B \xrightarrow{e} C \right)_{\mathbb{D}} \xrightarrow{\alpha} \left(A^{\alpha} \overset{\rightsquigarrow}{\underset{\rightsquigarrow}{\vdots}} B^{\alpha} \overset{\rightsquigarrow}{\underset{\rightsquigarrow}{\vdots}} C^{\alpha} \right)_{\mathbb{P}\xrightarrow{L}}$$

$$(d^{\alpha}_{\lambda})_{L} \qquad (e^{\alpha}_{\lambda})_{L}$$



Generated global dynamics

---

relax !

sub-functorial dynamics

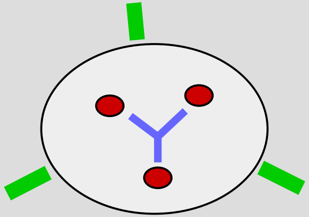
inclusion properties

Sub-functorial dynamics

$$\mathbb{D}:\; A \xrightarrow{d} B \xrightarrow{e} C \qquad \xrightarrow{\alpha} \qquad \mathbb{P}^{\overset{L}{\to}}:\; A^\alpha \rightsquigarrow B^\alpha \rightsquigarrow C^\alpha$$

$$(d^\alpha_\lambda)_L \qquad (e^\alpha_\lambda)_L$$

$$\forall \lambda \in L, \begin{cases} (Id_A)^\alpha_\lambda \subseteq Id_{A^\alpha} \\ \text{and} \\ e^\alpha_\lambda \circ d^\alpha_\lambda \subseteq (e \circ d)^\alpha_\lambda \end{cases}$$



Generated global dynamics

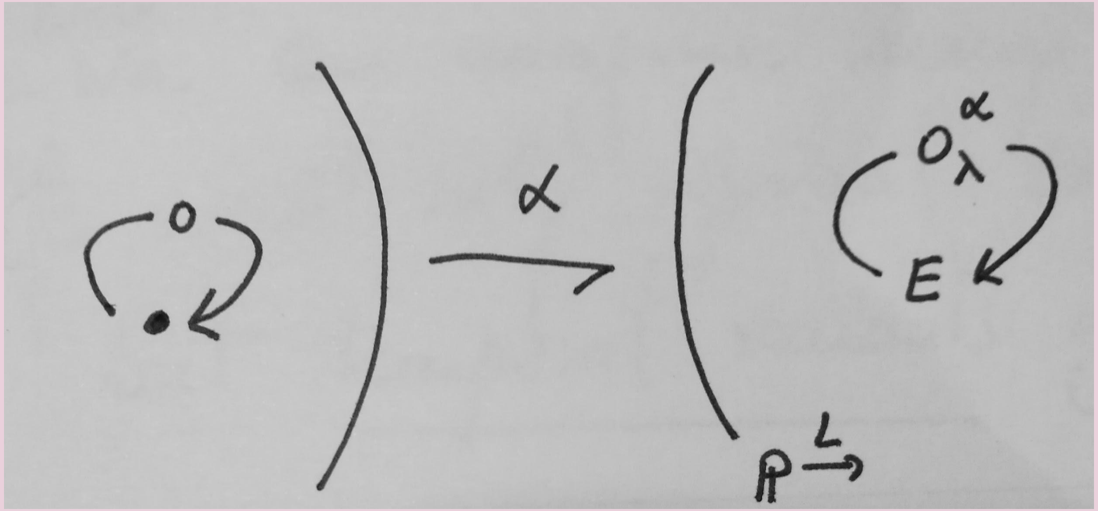
relax !

sub-functorial dynamics

inclusion properties

open timeless dynamics

Open timeless dynamics



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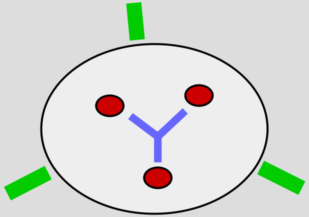

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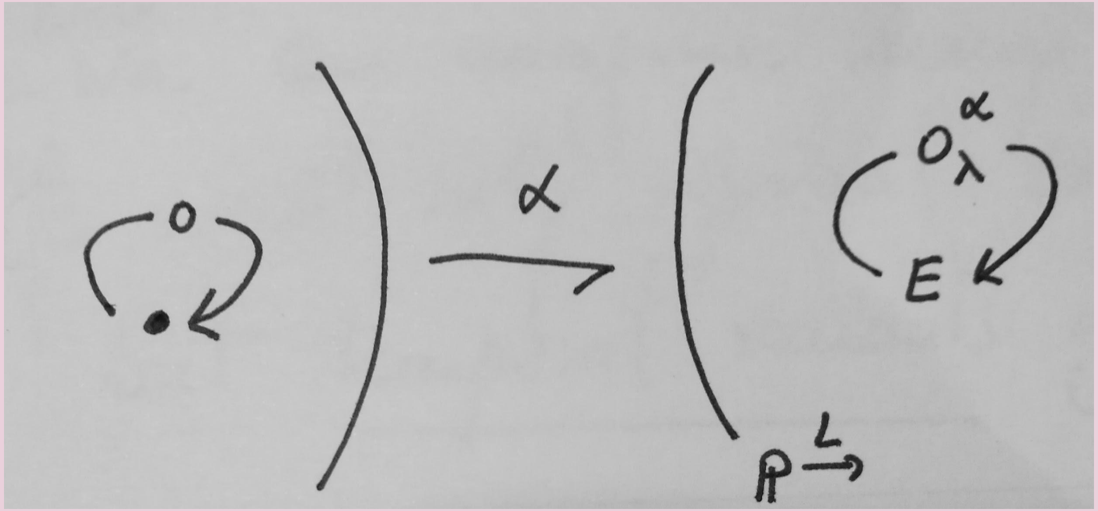

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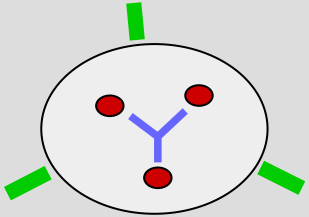
$$O^\alpha_\lambda \subseteq \mathrm{Id}_E$$



Generated global dynamics

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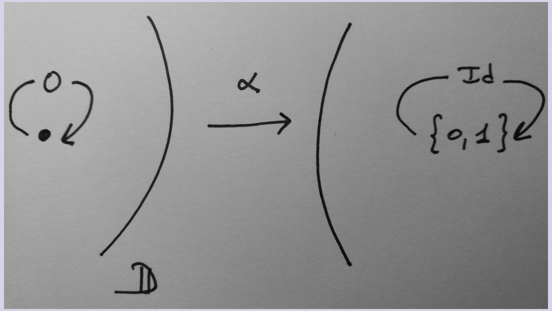
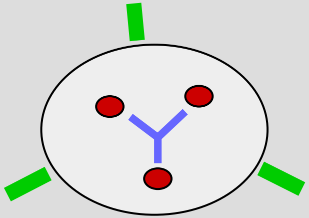
open timeless dynamics

a borromean interaction

An example of an interaction between timeless dynamics



a "zero-step game of life" individual cells is a timeless deterministic dynamic.



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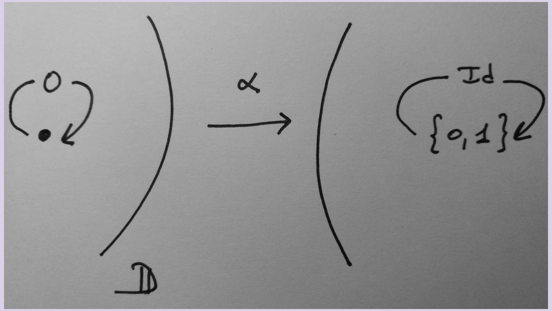

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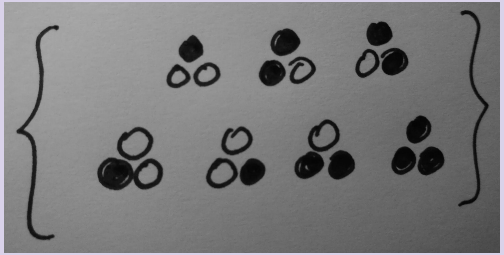
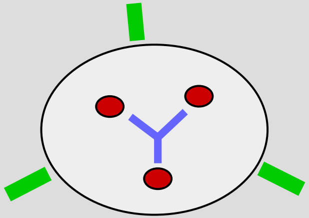


The interaction between 3 such dynamics such that at least one of their realisations is “alive”...



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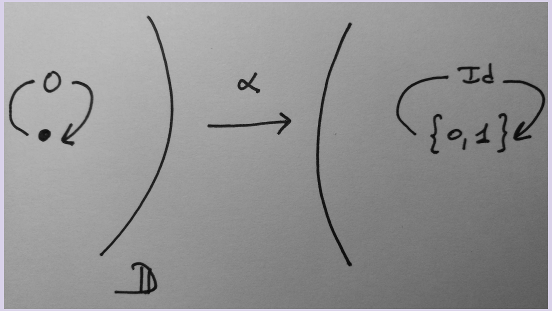

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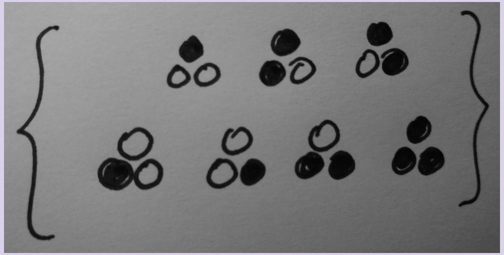

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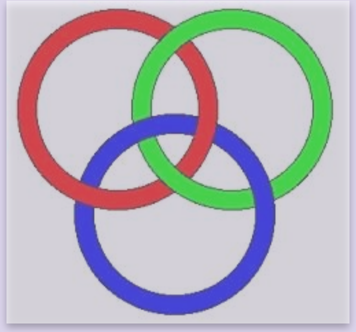
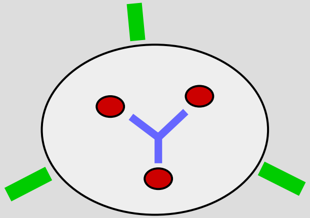


has a borromean connectivity



Generated global dynamics

---

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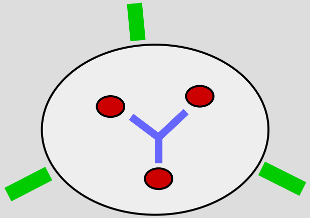
*(an example of a **normal** borromean interaction between three timeless open dynamics)*

$$\left(\circlearrowleft^{0} \bullet \right) \xrightarrow{\alpha} \left(\{0,1\} \overset{O^{\alpha}_{\lambda}}{\rightrightarrows} \{0,1\} \right) \quad \text{with} \quad \begin{cases} O^{\alpha}_{a} = Id \\ O^{\alpha}_{b} = \begin{cases} 0 \mapsto \phi \\ 1 \mapsto 1 \end{cases} \end{cases}$$

$$\mathbb{P} \xrightarrow{\{a,b\}}$$

Then the functional interaction defined by $|R| = \left\{ \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ \sigma_1 & \sigma_2 & \sigma_3 \end{pmatrix} \right.$ such that

$$\forall i \in \{1,2,3\}, \left| \begin{array}{l} \sum_{j \neq i} \sigma_j = 0 \Rightarrow \lambda_i = b \\ \text{otherwise } \lambda_i = a \end{array} \right. \quad \left. (\text{and } \sigma_i \in \varphi_{\lambda_i}) \right\} \text{ is borromean}$$



Generated global dynamics

relax !

sub-functorial dynamics

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The theorem

The fundamental theorem

Any interactive family of subfunctorial open dynamics generates some global subfunctorial open dynamics.

see § 3.1 and §3.2 in

arXiv.org > math > arXiv:1608.07938

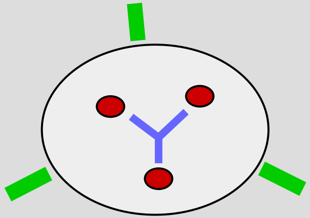
Mathematics > Dynamical Systems

Interacting Dynamics

Stéphane Dugowson (Quartz)

(Submitted on 29 Aug 2016 (v1), last revised 30 Aug 2016 (this version, v2))

https://arxiv.org/abs/1608.07938



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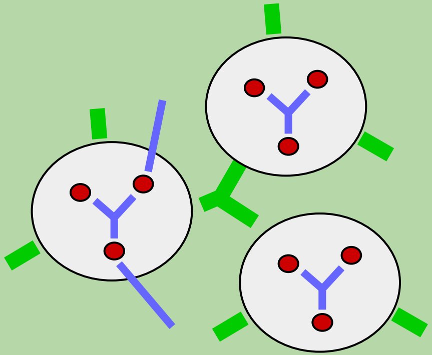

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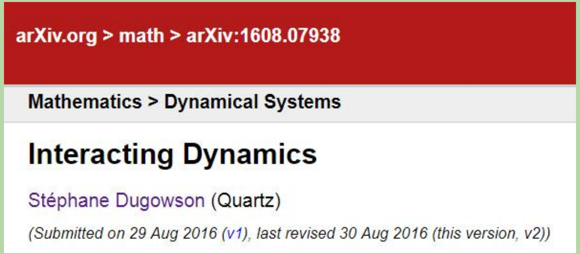
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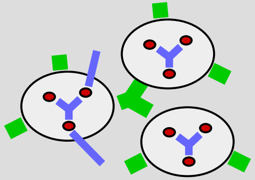
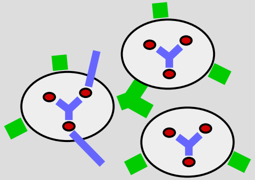


Conclusion

- The theory of open sub-functorial dynamics is a language to describe and **imagine** many kinds of interacting dynamics, each with its own temporality.



Conclusion

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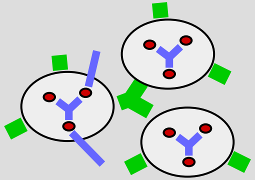
We can design many *temporal* variants of the game of life in which some cells could have

- continuous time,
- cyclic time,
- multi-dimensionnal time,
- non-monoidal time,
- “free will”,
- etc...



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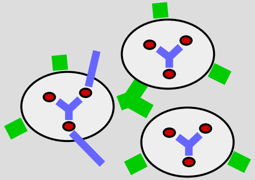
- It would be interesting to study the connections with some other theories concerning open systems, like the one developed by D. Spivak, C. Vasilakopoulou and P. Schultz.



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- It would be interesting to study the connections with some other theories concerning open systems, like the one developed by D. Spivak, C. Vasilakopoulou and P. Schultz.

- Some connectivity aspects still have to be developed.

References

S. Dugowson

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A. (Bastiani) Ehresmann

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D. Spivak, C. Vasilakopoulou and P. Schultz

Dynamical Systems and Sheaves, sept 2016, arXiv.


Thank you for your attention !