# Compositional Design and Tasking of Networks

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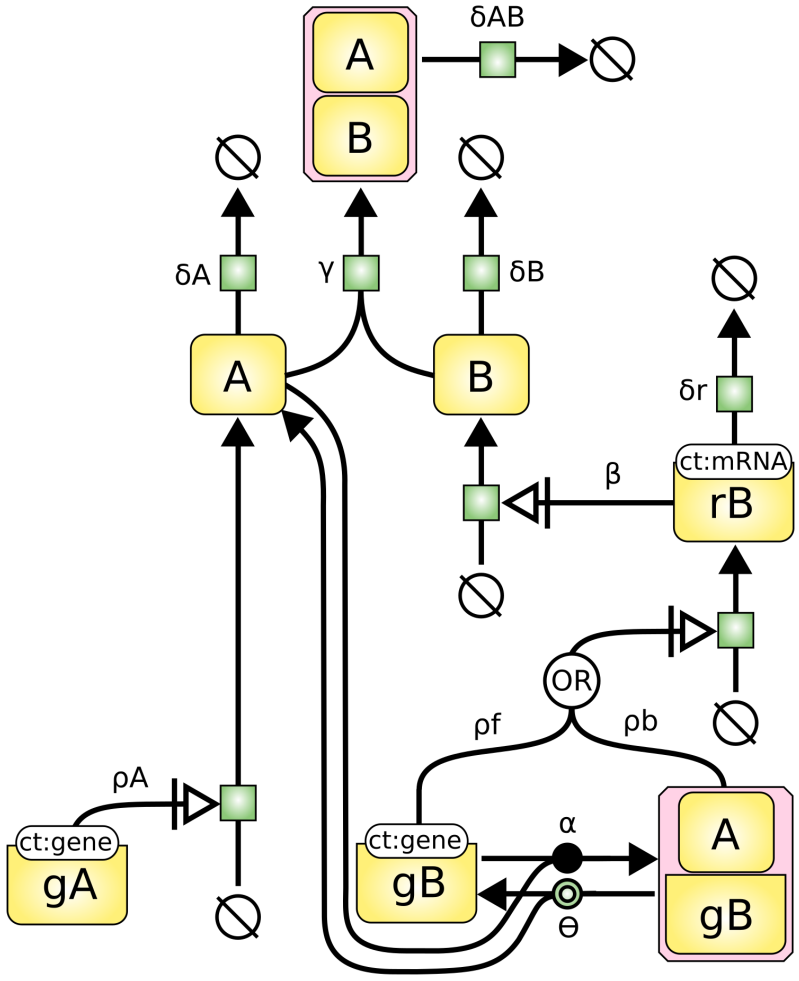
Complex systems are often studied and designed using *networks*:



Categories and operads are uniquely suited to describing networks and ways of composing them.

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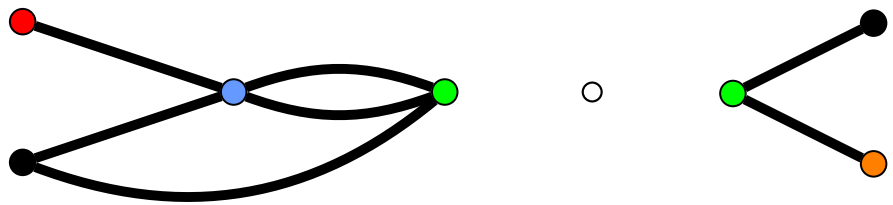
Networks can be connected ‘end to end’, in series:



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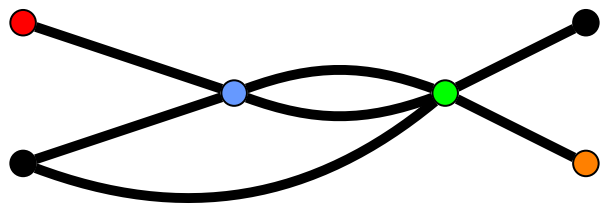
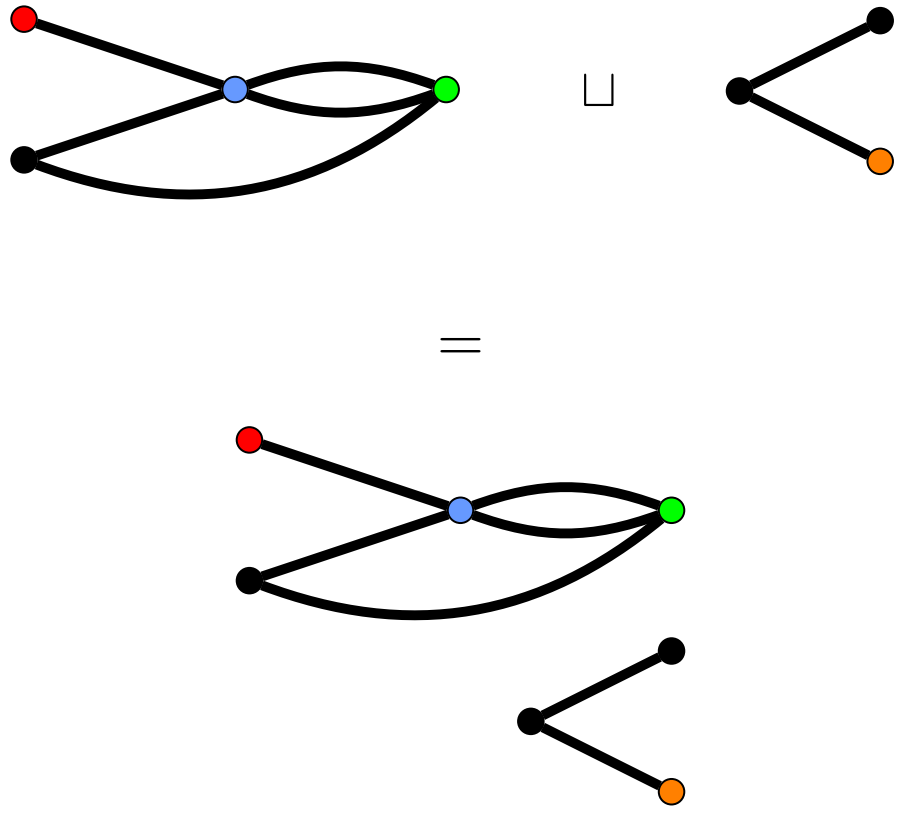


Networks can also be set ‘side by side’, in parallel:



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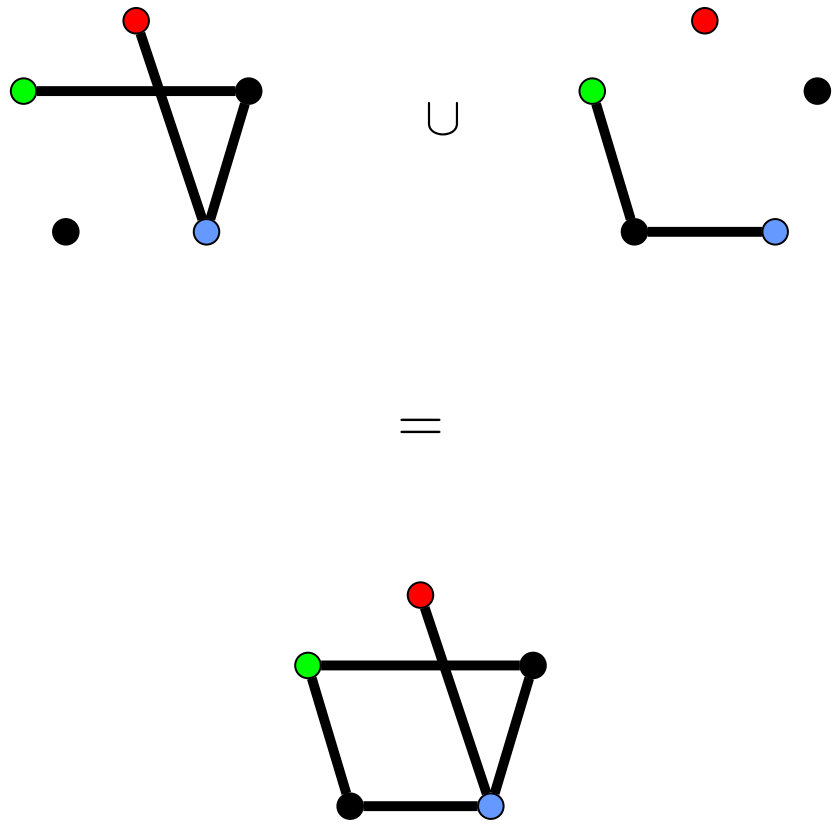
In the Complex Adaptive System Composition and Design Environment project, I'm working with Metron Scientific Solutions to build networks using another form of composition: *overlaying*.

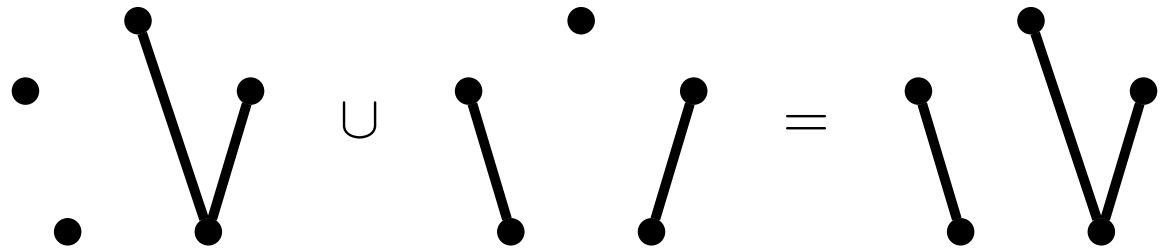
The hope is that new ideas from category theory can give new ways to design complex systems.

Sometimes we can ‘overlay’ two networks with the same vertices. For example,



There are many kinds of networks, and many possible rules for overlaying them.

Simple graphs:



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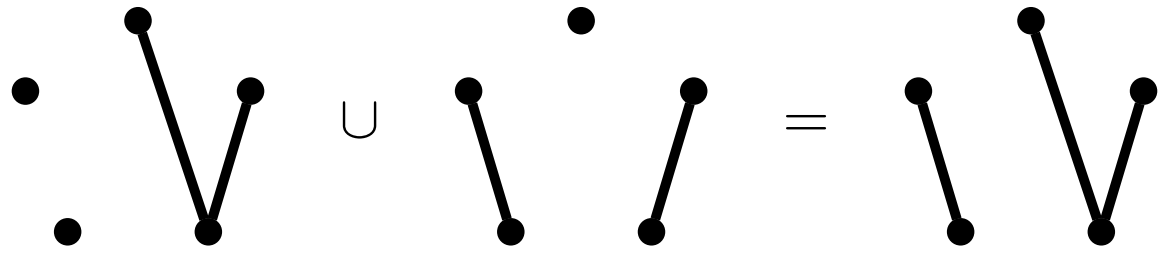
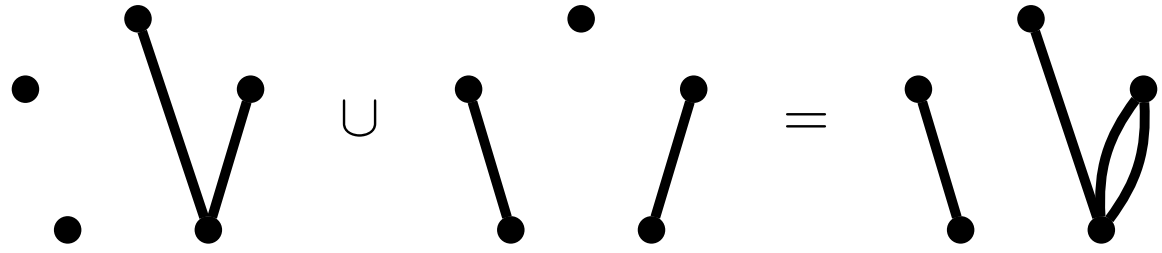
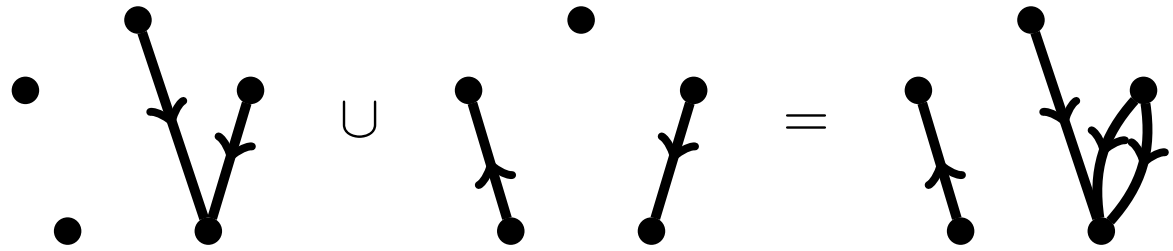


Multigraphs:



Directed multigraphs:



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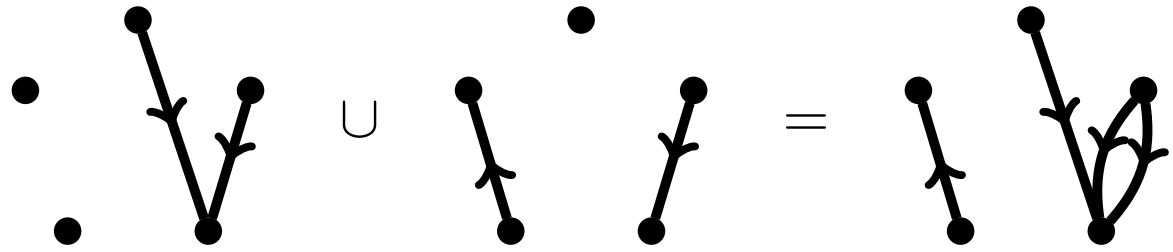
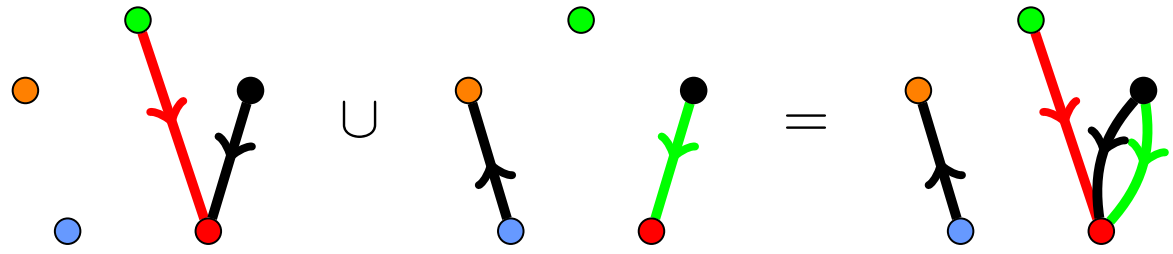


Directed multigraphs with colored edges and nodes:



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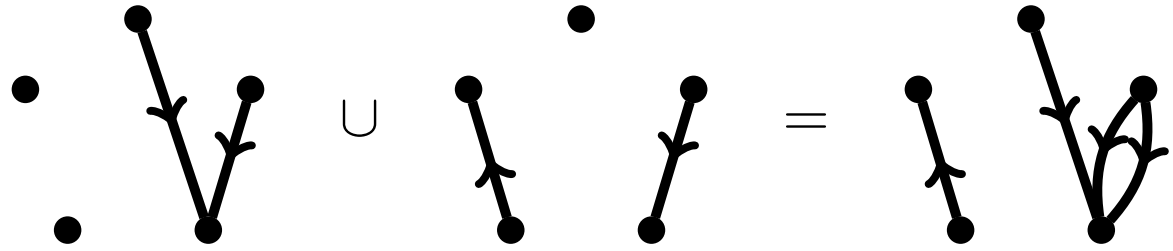

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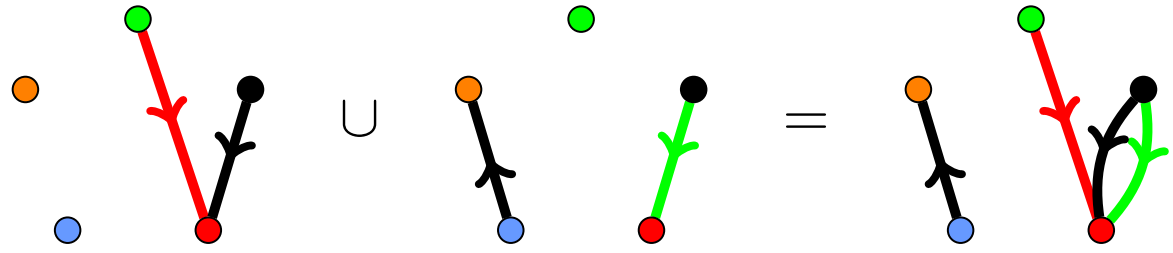

... and so on!

We should handle all these kinds of networks in a unified way.

Define a **network model** to be a symmetric lax monoidal functor

$$F : S(C) \to \mathrm{Mon}$$

where $S(C)$ is the free symmetric monoidal category on some set $C$ of **vertex colors**, and Mon is the category of monoids.

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For example: if $C = 1$, $S(C)$ is the groupoid of finite sets and bijections. For any finite set $X$, $F(X)$ is the set of networks having $X$ as their set of vertices. This is a monoid, so we can **overlay** networks:

$$\cup \colon F(X) \times F(X) \to F(X)$$

Since $F$ is lax monoidal we can also set networks 'side by side':

$$\sqcup \colon F(X) \times F(Y) \to F(X + Y)$$

For any network model $F$ there is an operad $O_F$ whose operations are ways to assemble networks of type $F$.

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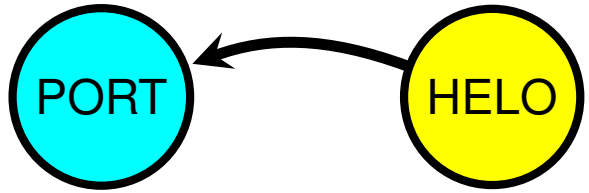
and much more. *Maps between algebras* let us design and task systems incrementally, adding a bit of detail at a time.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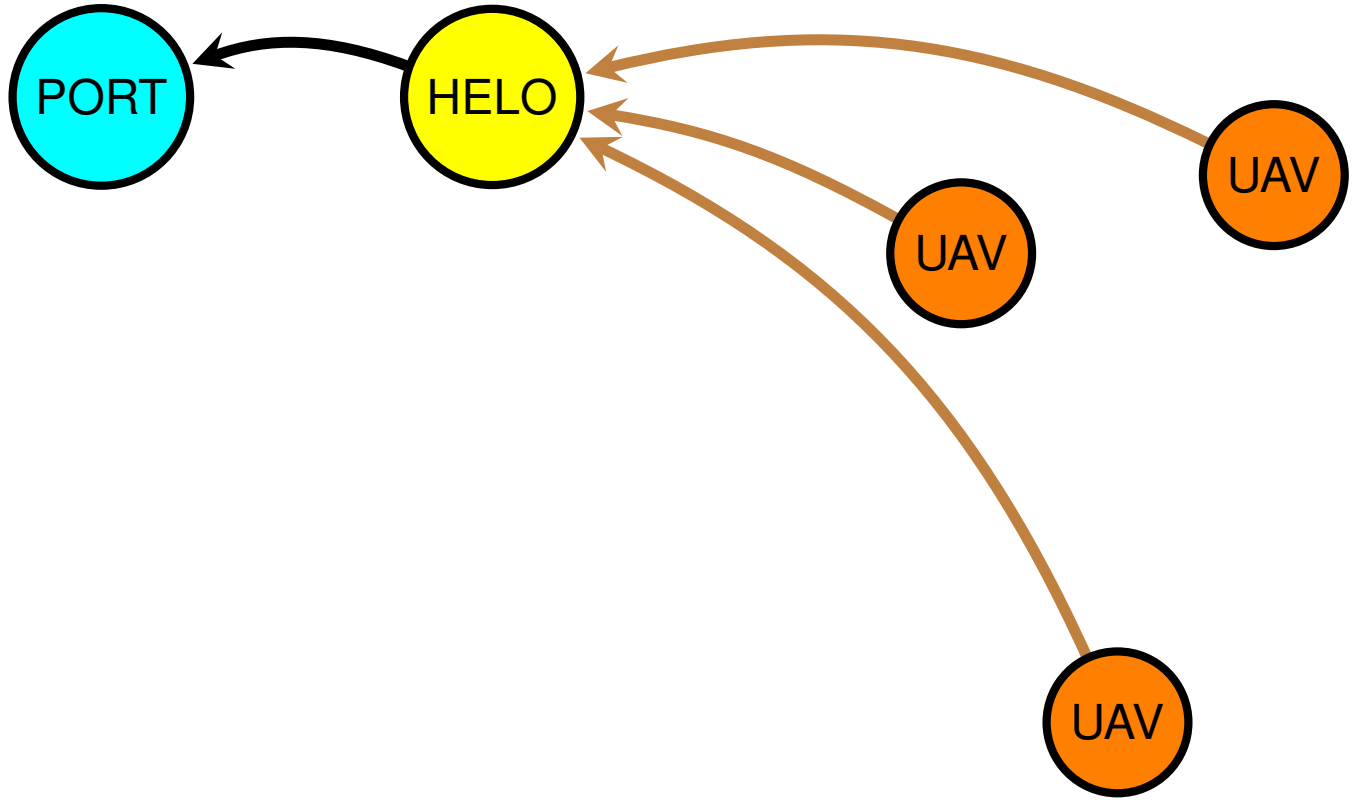
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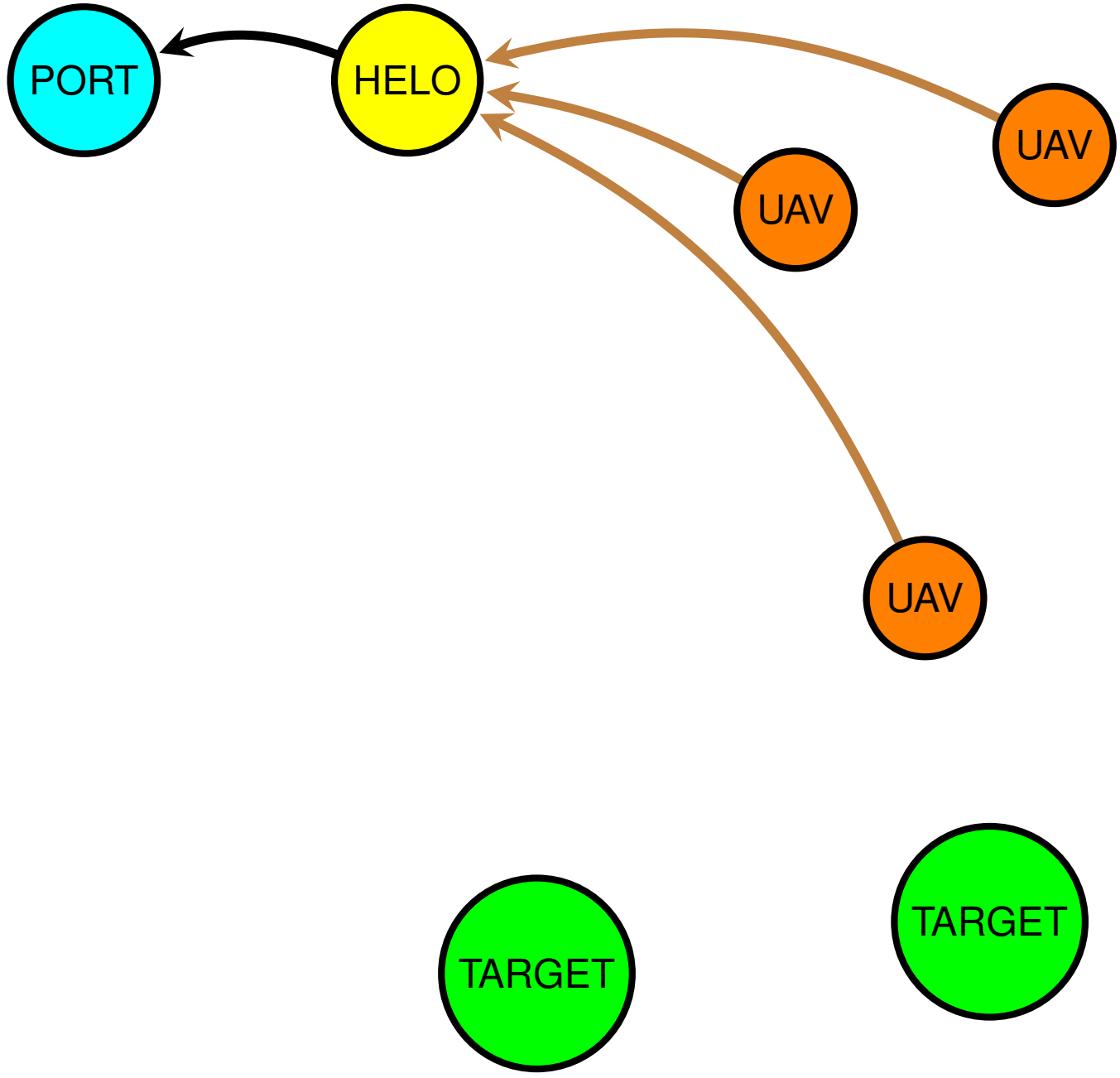
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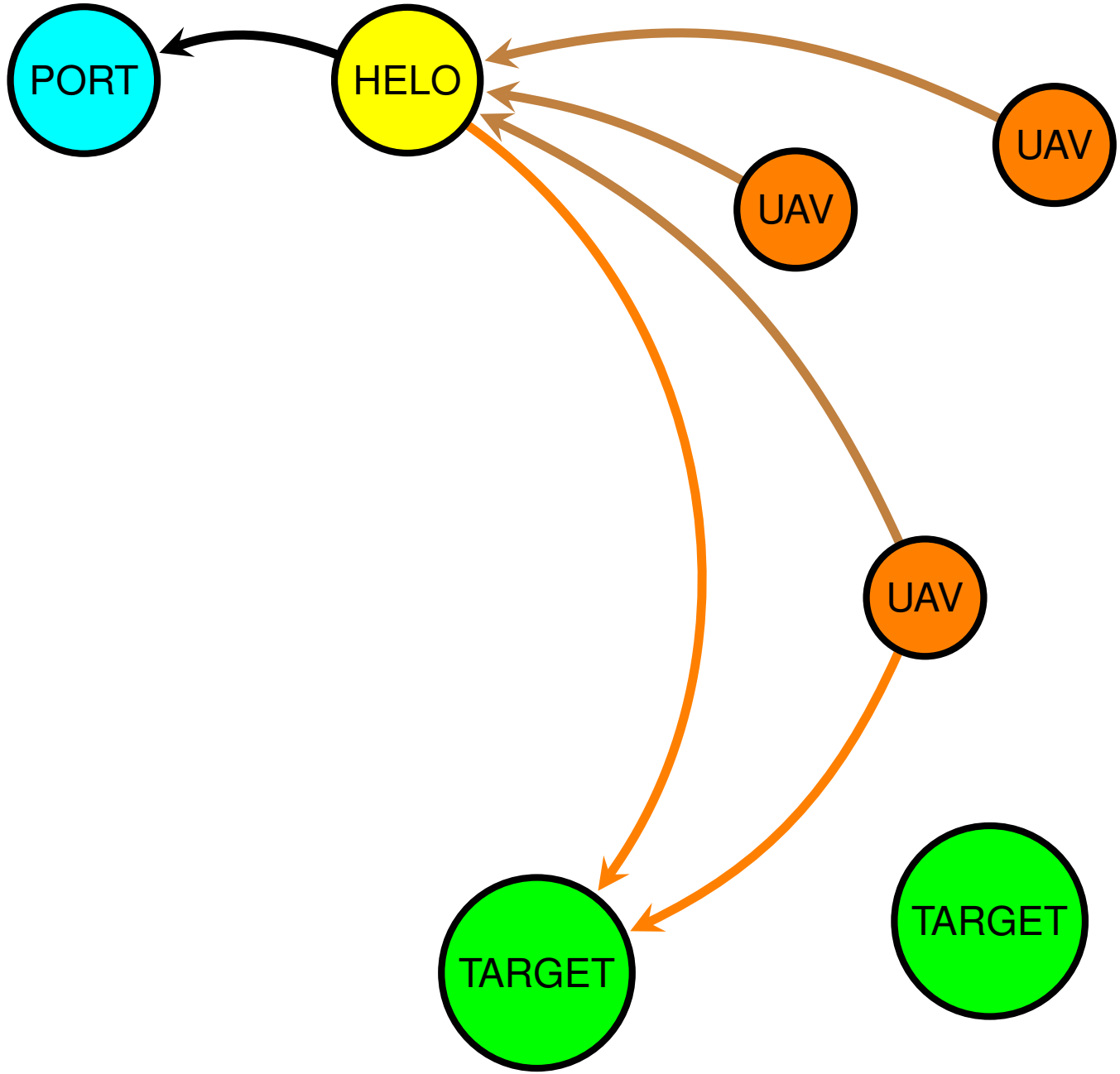
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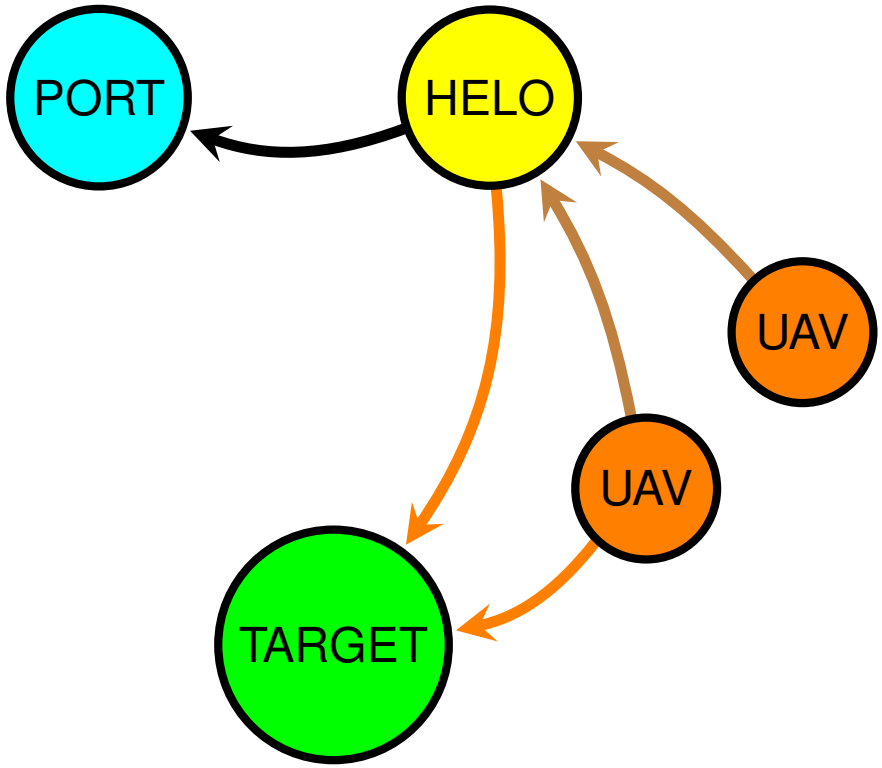
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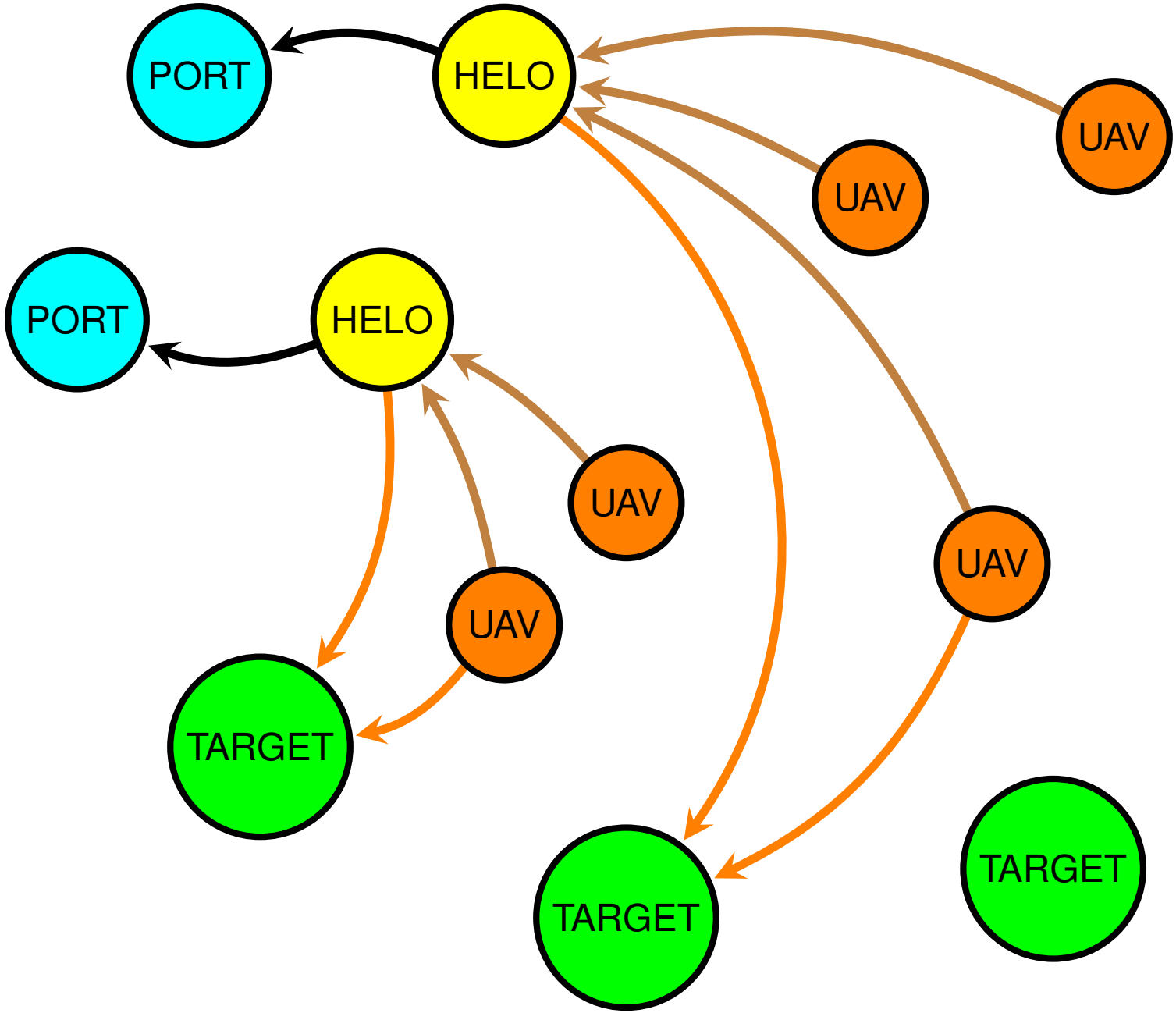
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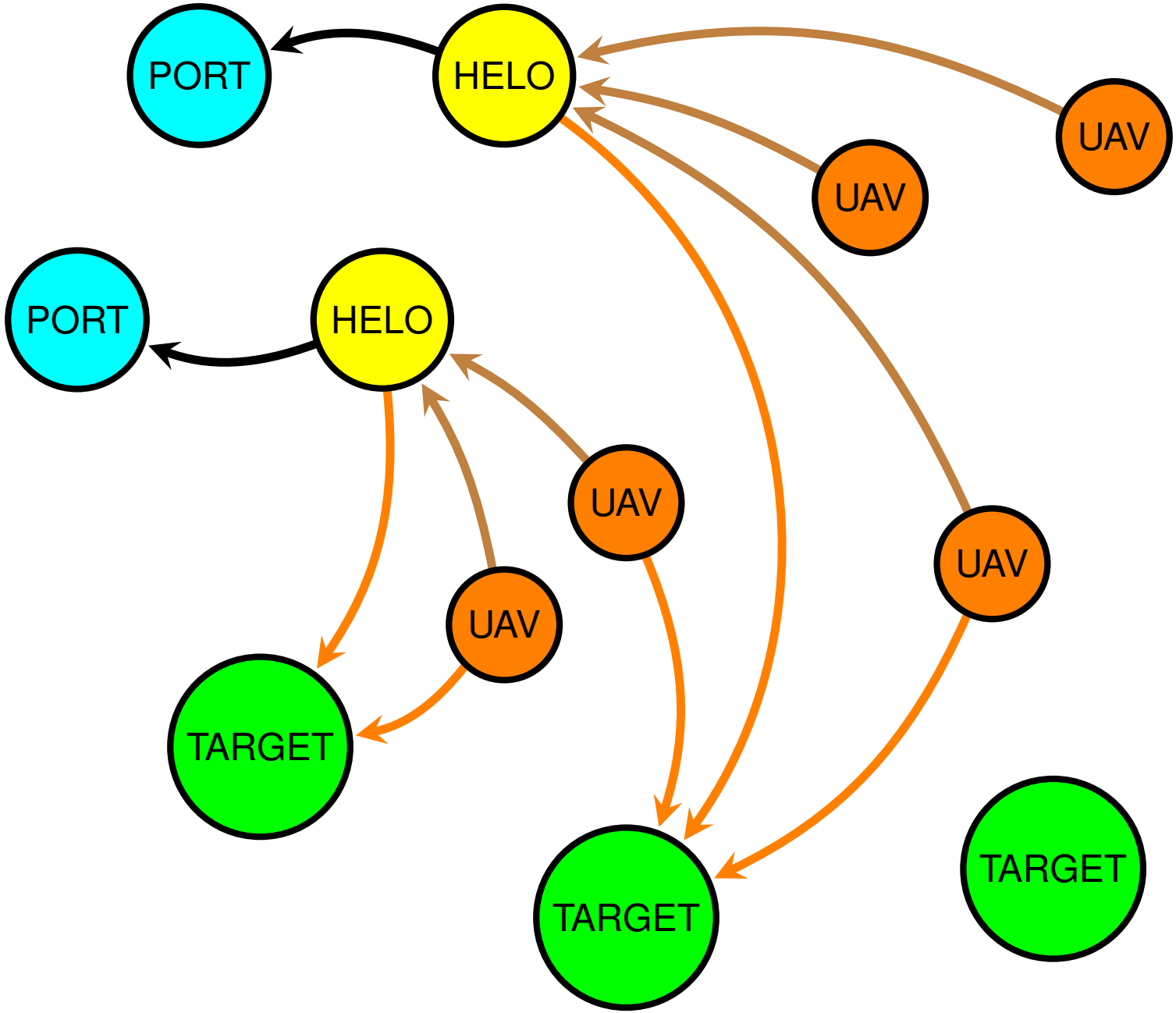
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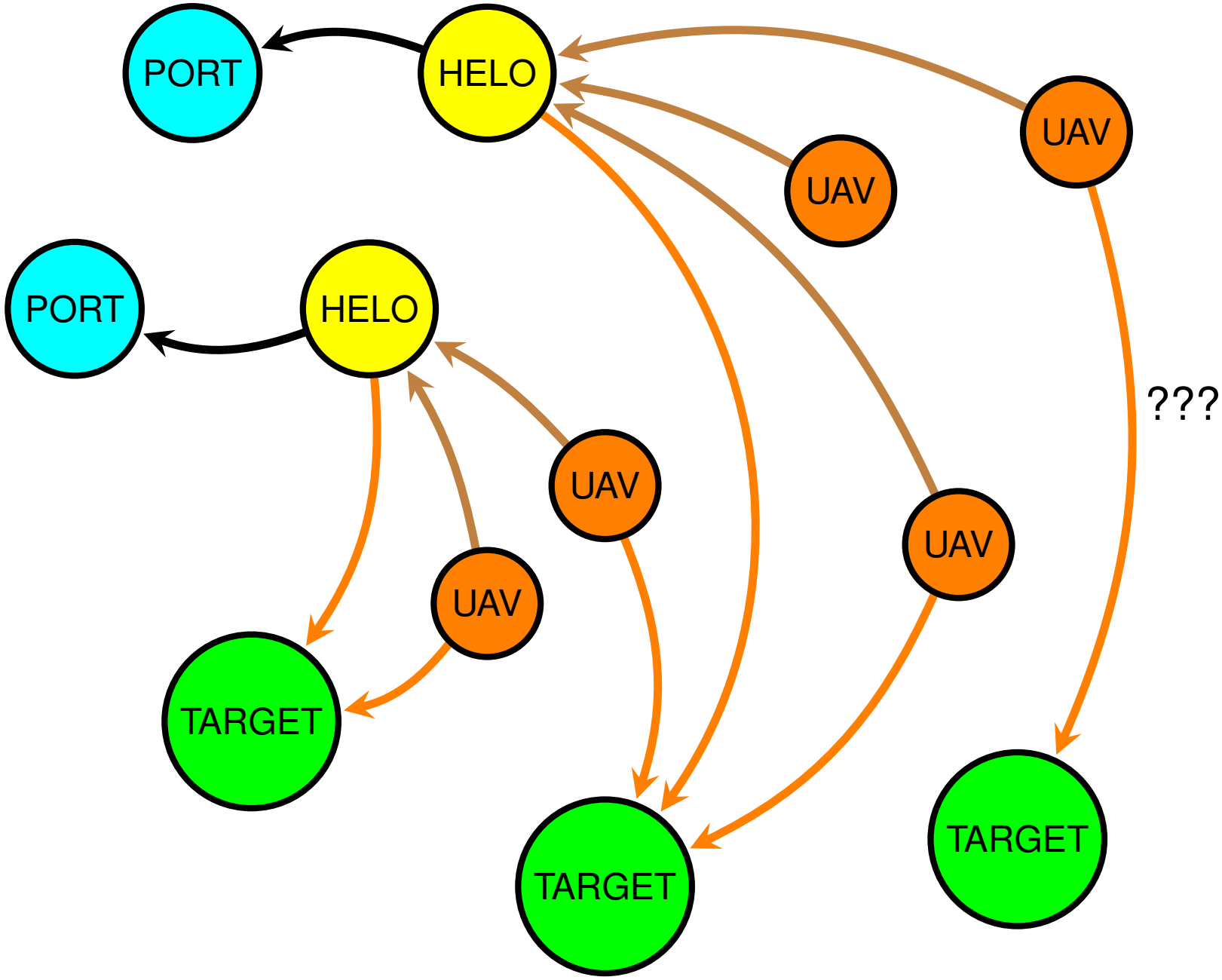
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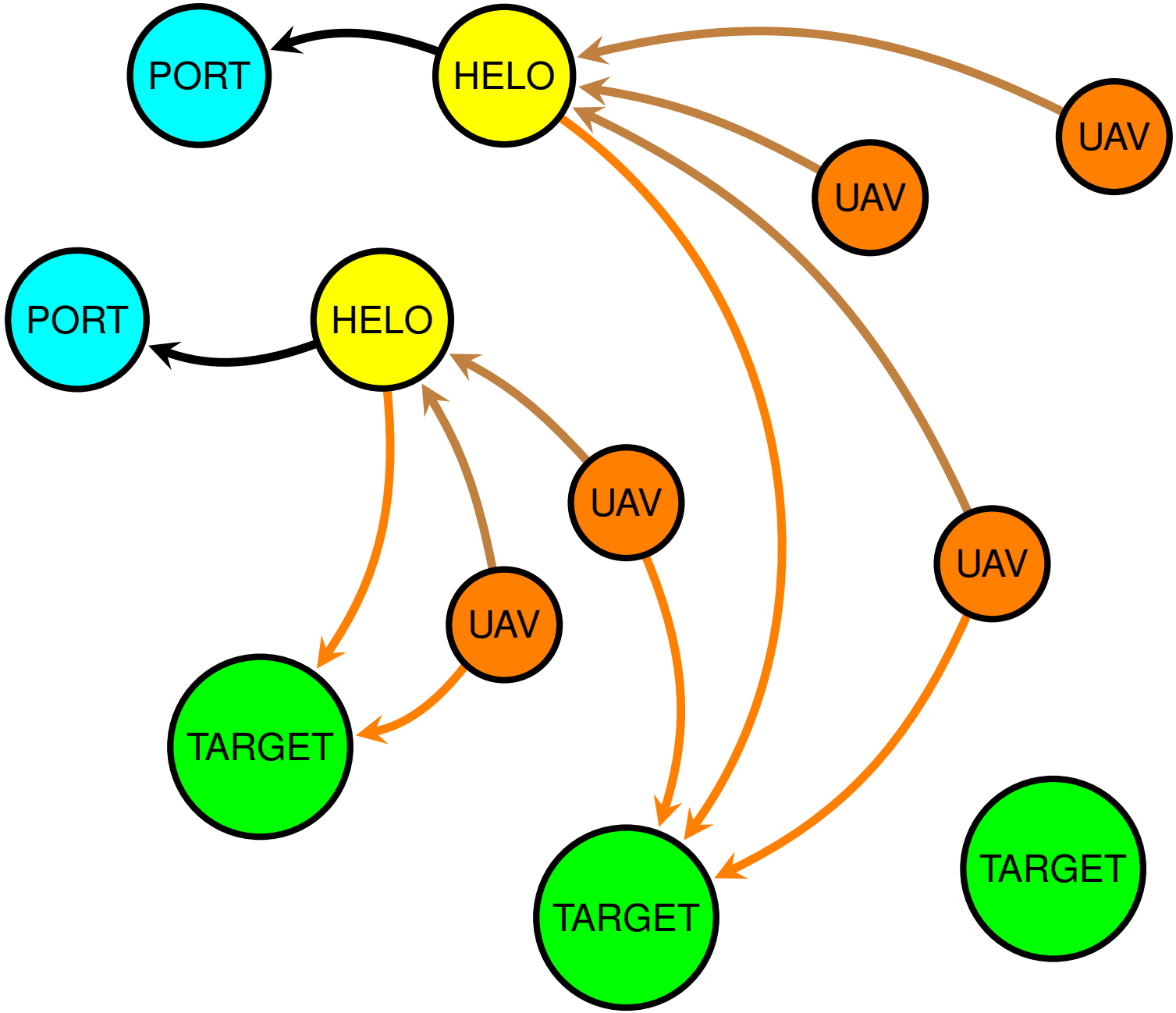
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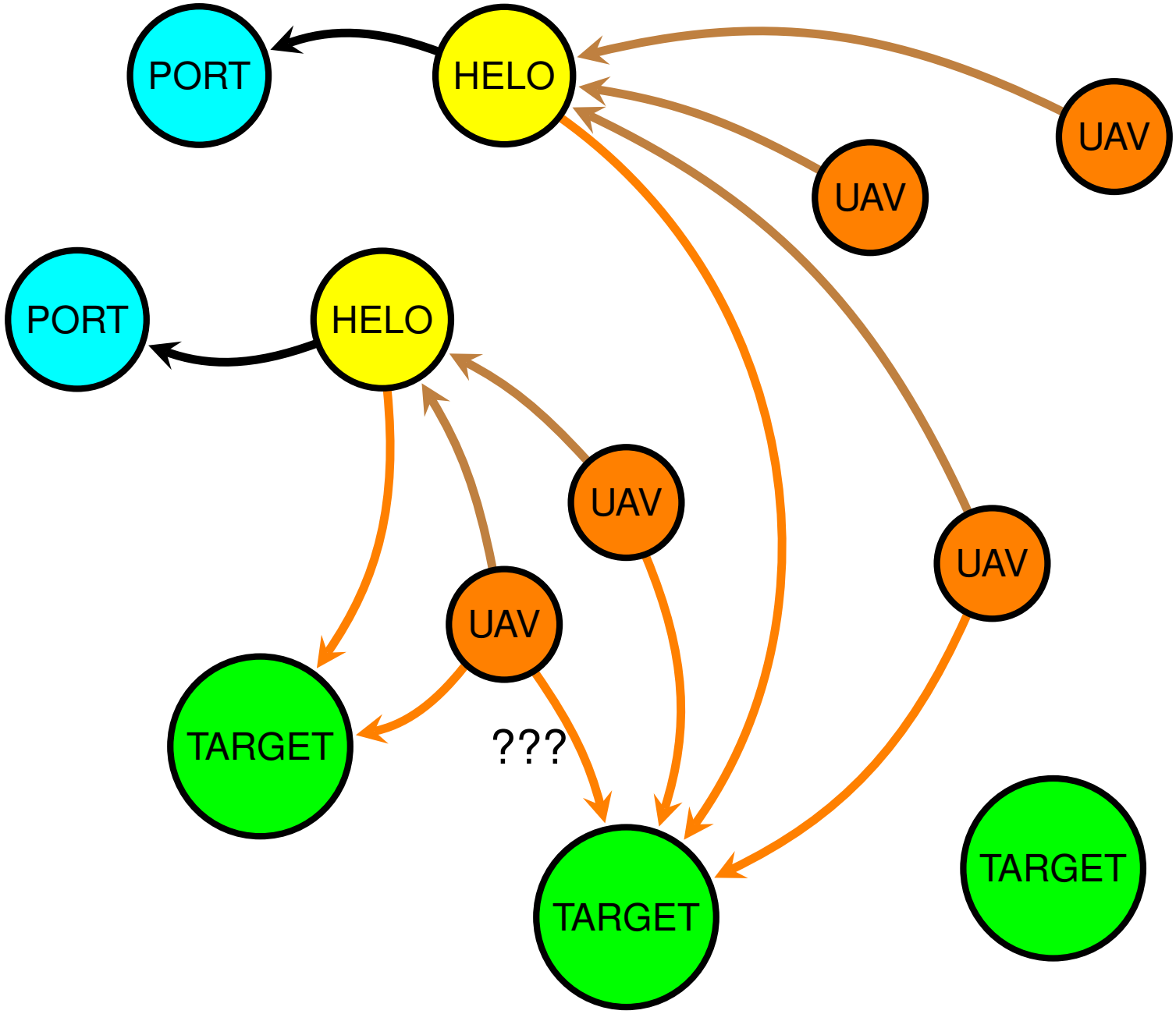
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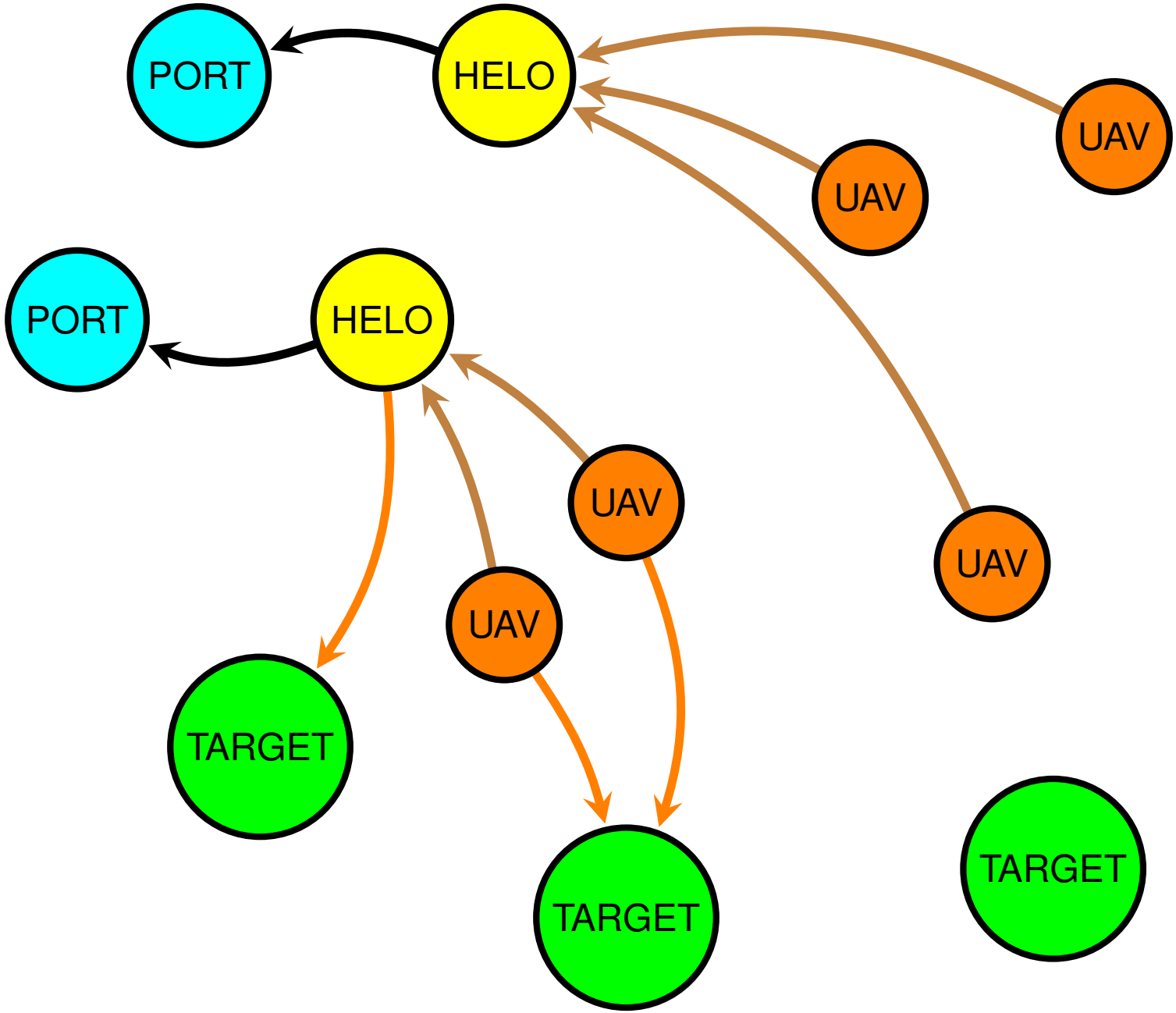
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For more see:

- John Baez, John Foley, Joseph Moeller and Blake Pollard, Network models: arXiv:1711.00037.

- John Baez, Complex adaptive system design, *Azimuth*.