Data Structures for Network Languages

Brendan Fong (MIT)

Category Theory Workshop
NIST
15 March 2018

Backprop as Functor

Brendan Fong (MIT), with David Spivak and Rémy Tuyéras

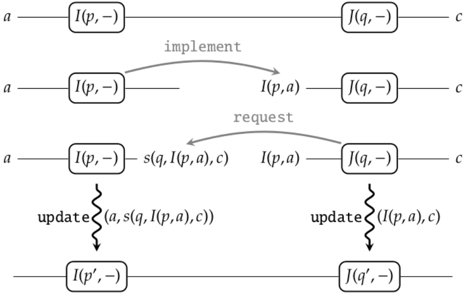
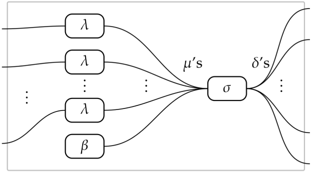
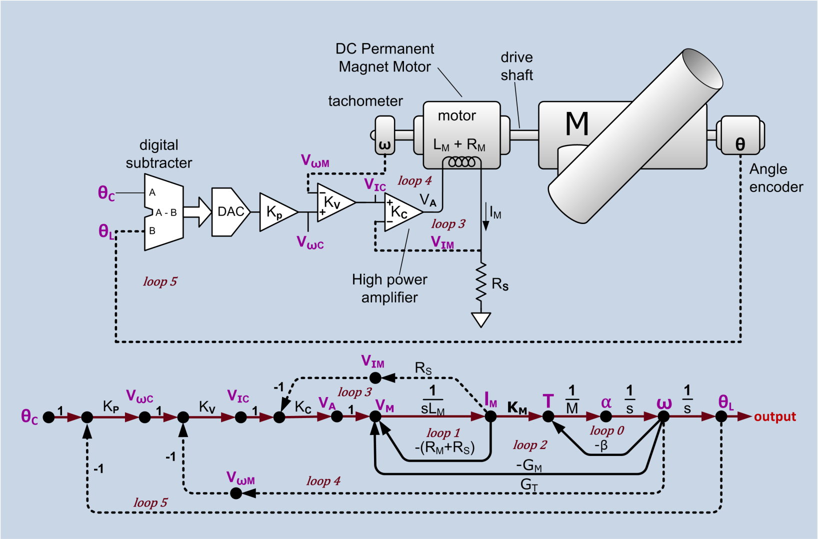
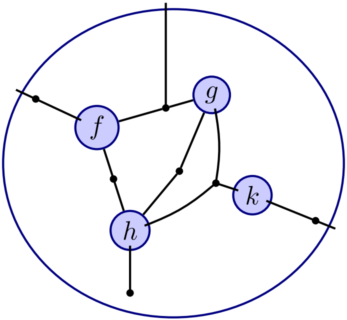


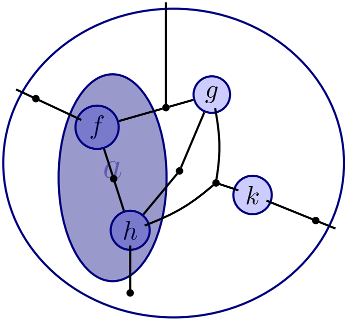
Figure 2. A request function allows an update function to be defined for the composite $J(q, I(p, -))$.





DC Permanent Magnet Motor
drive shaft
tachometer
motor
$L_M + R_M$
M
ω
θ
Angle encoder
digital subtracter
θ_C
θ_L
A
A - B
B
DAC
K_p
$V_{\omega M}$
$V_{\omega C}$
K_V
V_{IC}
K_C
V_A
loop 4
loop 3
I_M
V_{IM}
R_S
High power amplifier
loop 5
θ_C
1
K_P
$V_{\omega C}$
1
K_V
V_{IC}
1
-1
K_C
V_A
1
V_M
V_{IM}
R_S
loop 3
$\frac{1}{sL_M}$
I_M
K_M
T
$\frac{1}{M}$
α
$\frac{1}{s}$
ω
$\frac{1}{s}$
θ_L
output
loop 1
$-(R_M+R_S)$
loop 2
loop 0
$-\beta$
$-G_M$
G_T
$V_{\omega M}$
loop 4
-1
-1
loop 5


g
f
k
h


g
f
a
k
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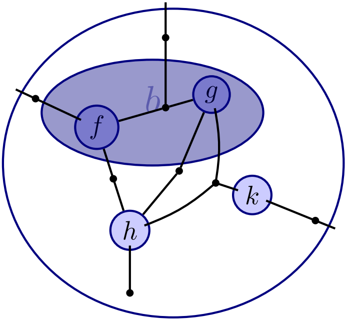
=


b
g
f
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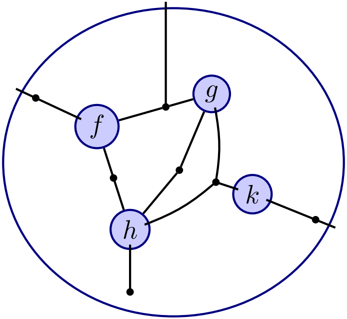

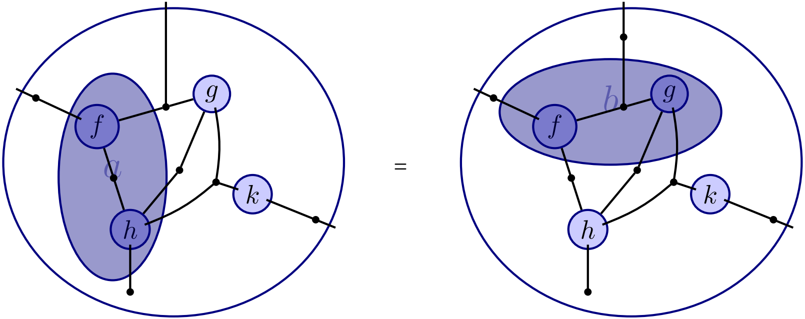

We'll call these hypergraph categories.

A data structure problem.

To specify a hypergraph category:

(i) list all systems f, g etc.

(ii) list the composition rule: for **all** systems arranged in **all** possible networks name the composite system.

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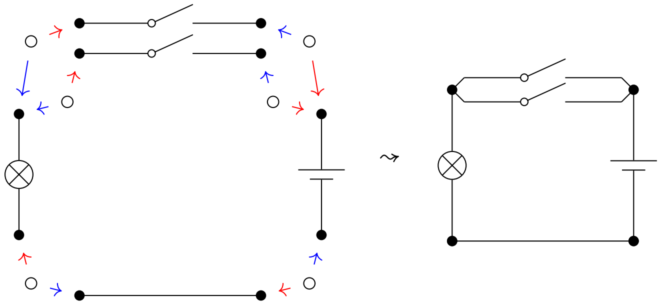
then check this data **coheres**.



This corresponds to taking the *colimit* of $\{\text{white } \circ\} \underset{g}{\overset{f}{\rightrightarrows}} \{\text{black } \bullet\}$

To specify a **decorated cospan** hypergraph category:

(i) list all systems f, g etc.

(ii) list how systems interact with functions.

then check this data forms a lax monoidal functor.

Universal constructions (colimits; a left Kan extension) take care of the rest.

For details, `http://brendanfong.com/`:

Decorated Cospans

Decorated Corelations

The Algebra of Open and Interconnected Systems

Seven Sketches in Compositionality

arXiv.org > math > arXiv:1803.05316

Mathematics > Category Theory

Seven Sketches in Compositionality: An Invitation to Applied Category Theory

Brendan Fong, David I Spivak

(Submitted on 14 Mar 2018)

Chapters:

1. Generative effects: Posets and adjunctions
2. Resources: Monoidal posets and enrichment
3. Databases: Categories, functors, and universal constructions
4. Co-design: Profunctors and monoidal categories
5. Signal flow graphs: Props, presentations, and proofs
6. **Circuits: Hypergraph categories and operads**
7. Logic of behavior: Sheaves, toposes, and languages