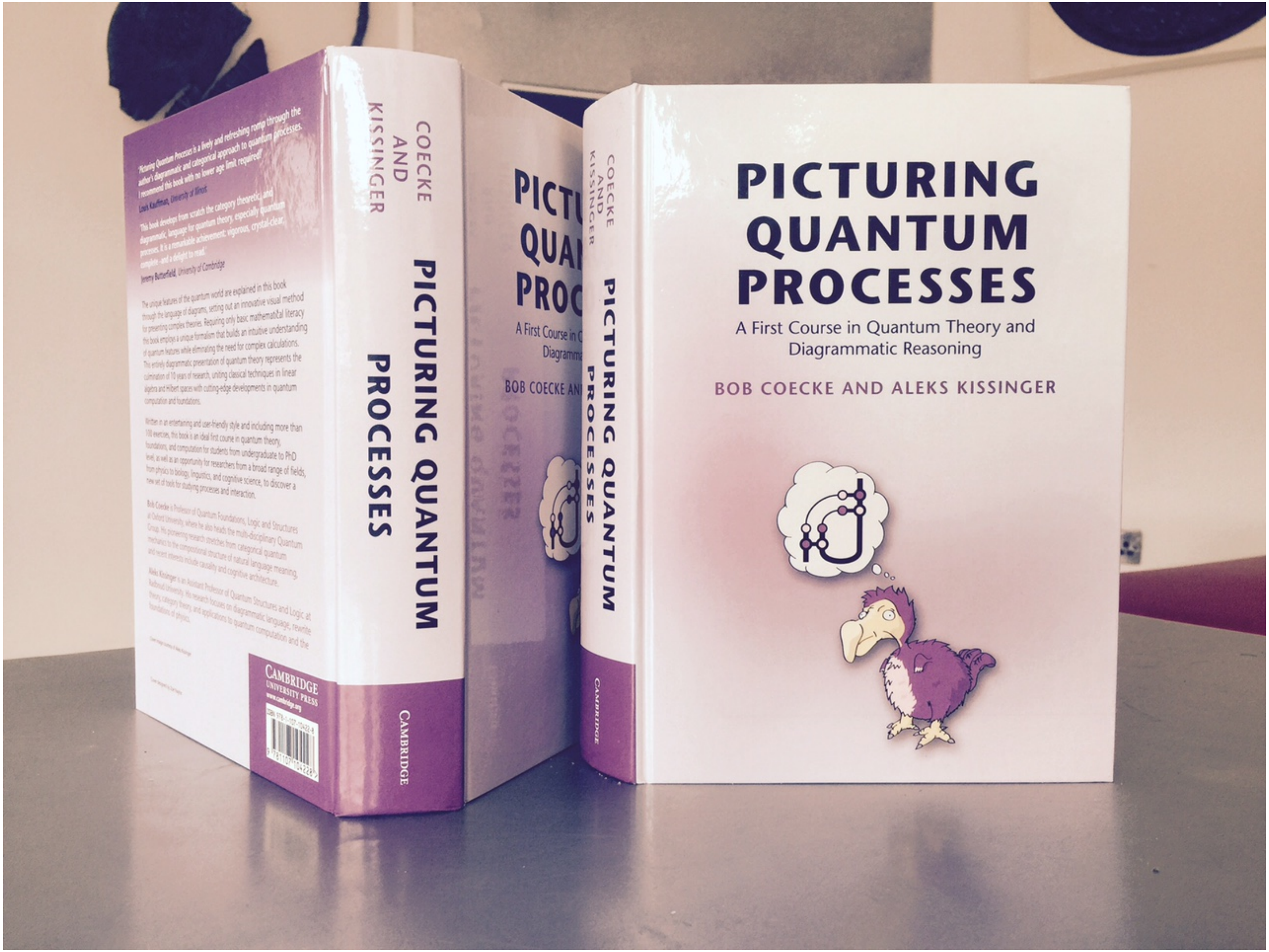
Today's dense menu:

- pictorial formalism for **quantum systems**



Today's dense menu:

- pictorial formalism for **quantum systems**
- **natural language meaning** composition

QUANTUM LINGUISTICS Leap forward for artificial intelligence

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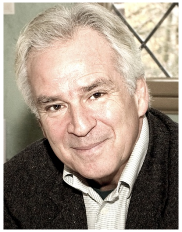
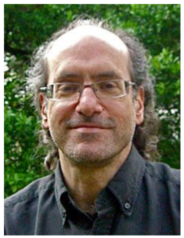
- compositional **cognition**



$\hookrightarrow$ **<u>ConvRel</u>**



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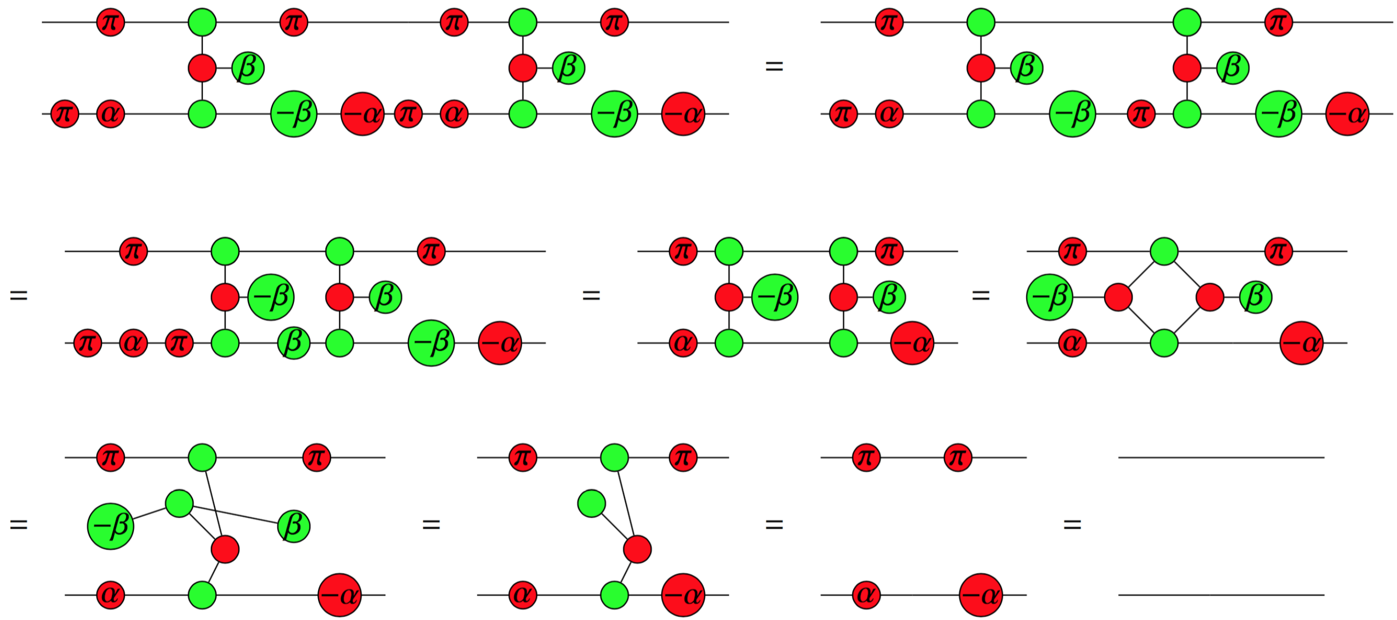
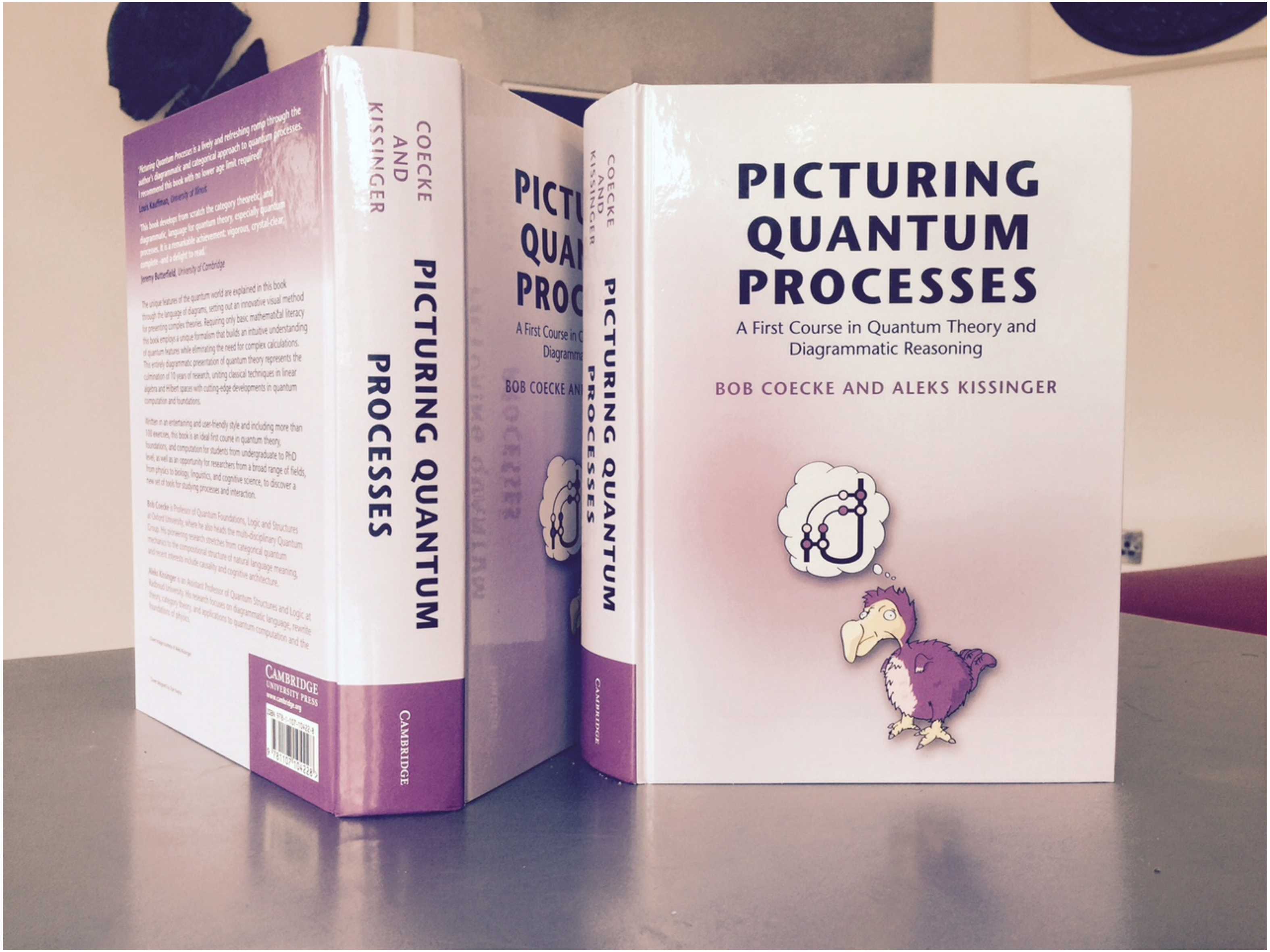
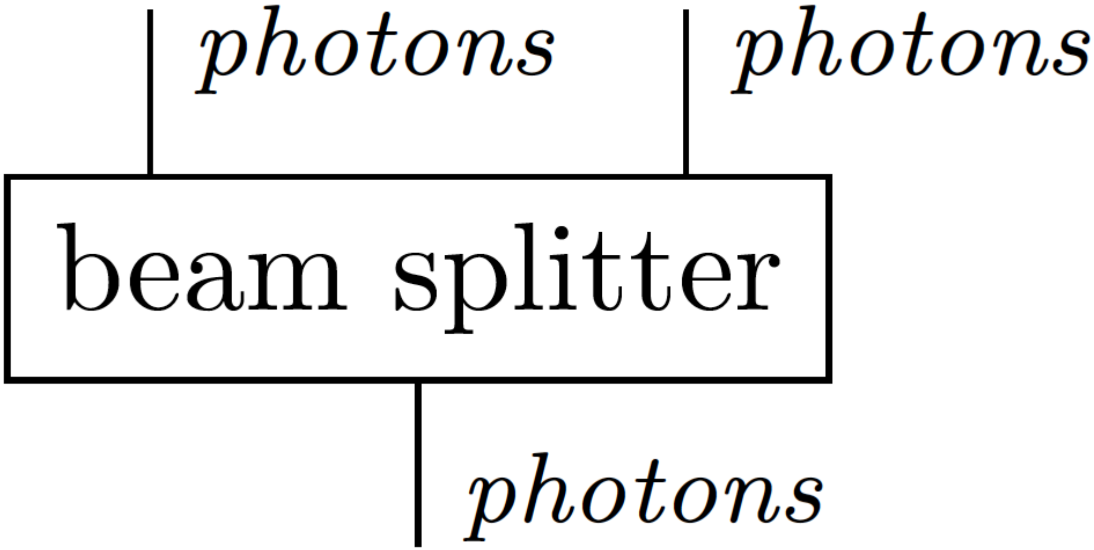
- compositional **cognition**
- efficient **quantum compilation**

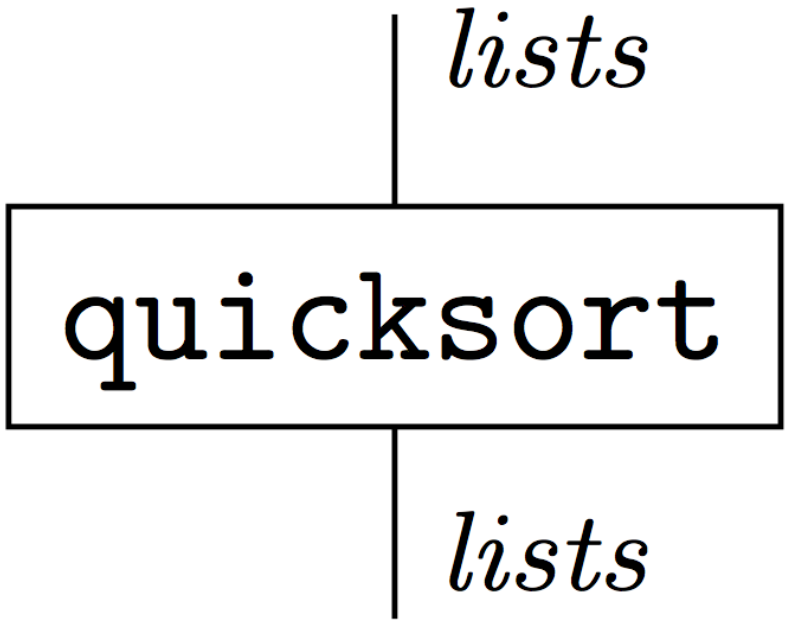


process theories and quantum

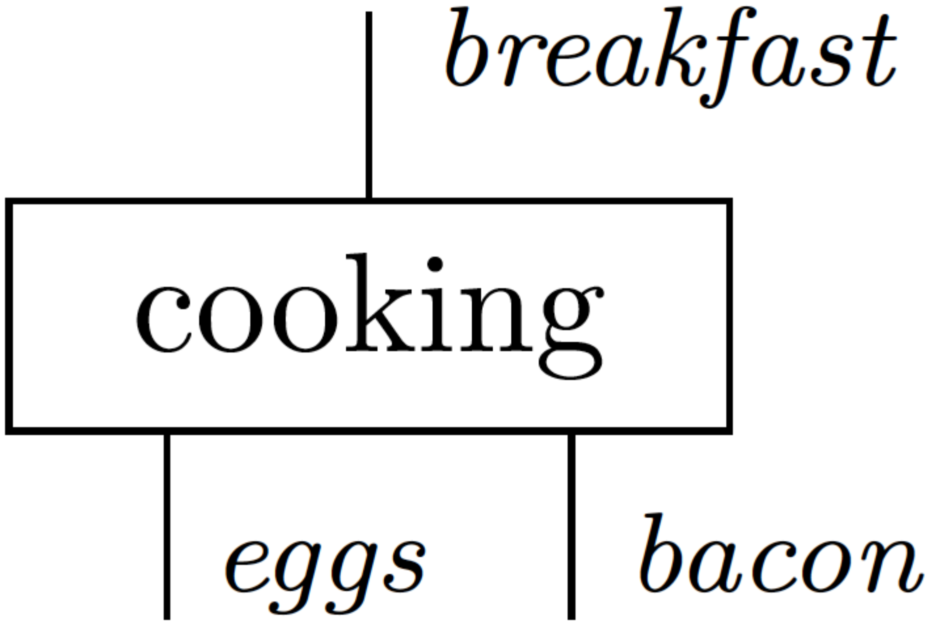

COECKE
AND
KISSINGER
PICTURING QUANTUM
PROCESSES
CAMBRIDGE
CAMBRIDGE
UNIVERSITY PRESS
PICTURING
QUANTUM
PROCESSES
A First Course in Quantum Theory and
Diagrammatic Reasoning
BOB COECKE AND ALEKS KISSINGER



photons
photons
beam splitter
photons


lists
quicksort
lists

– processes as boxes and systems as wires –



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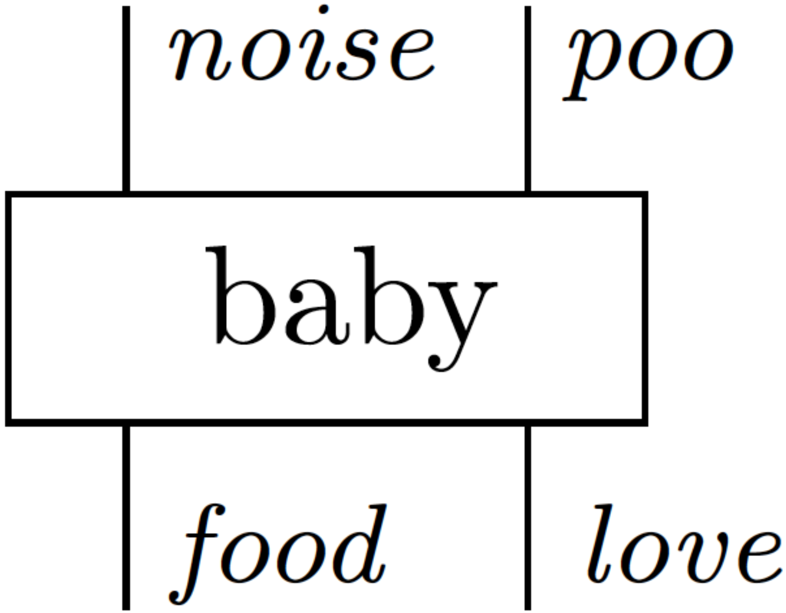
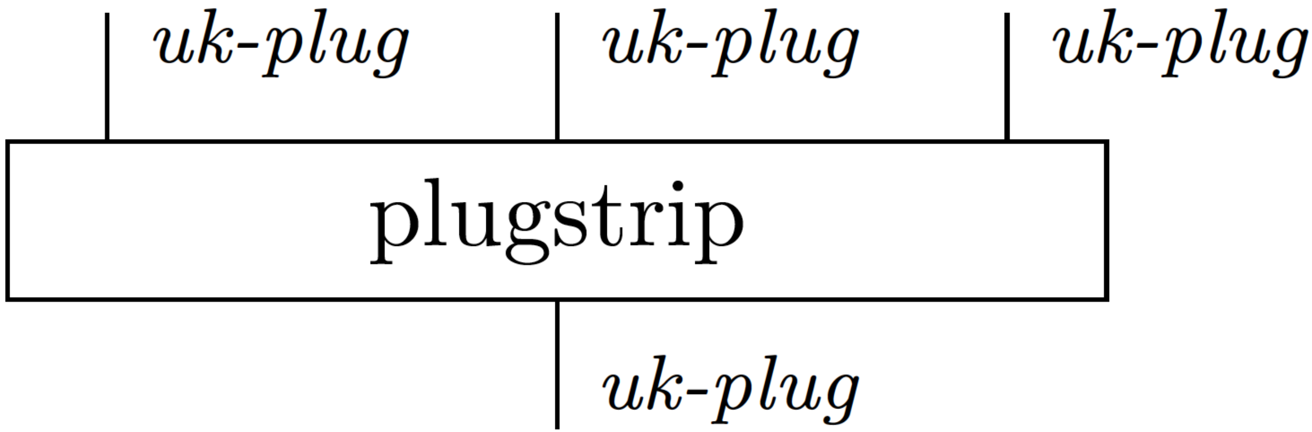
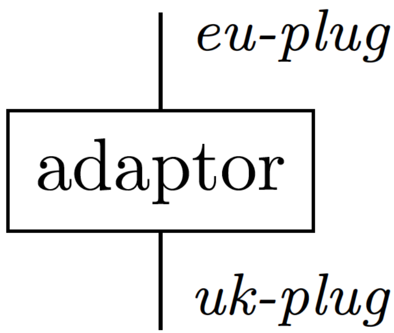
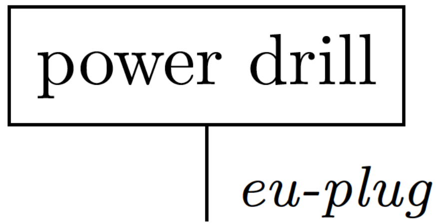


– composing processes –







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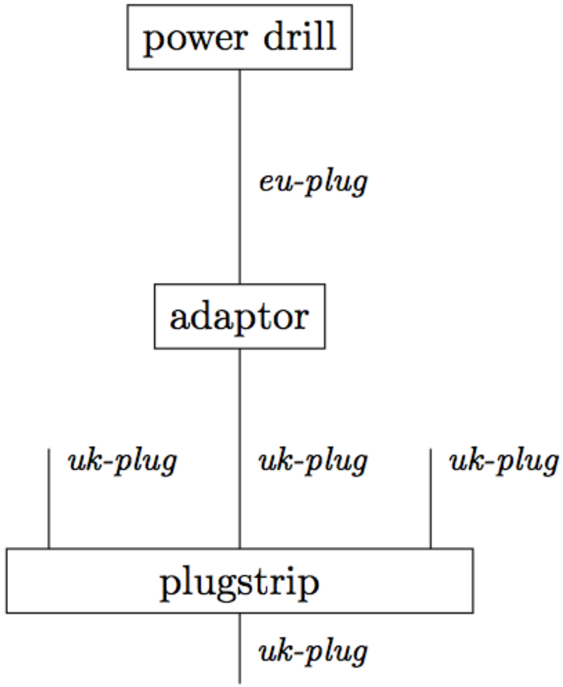
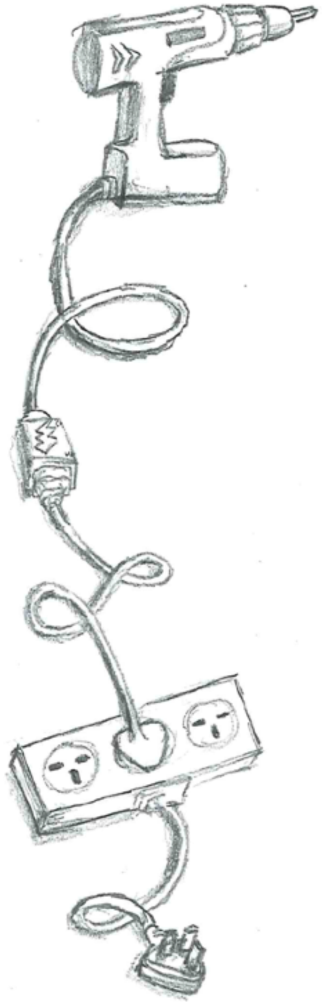
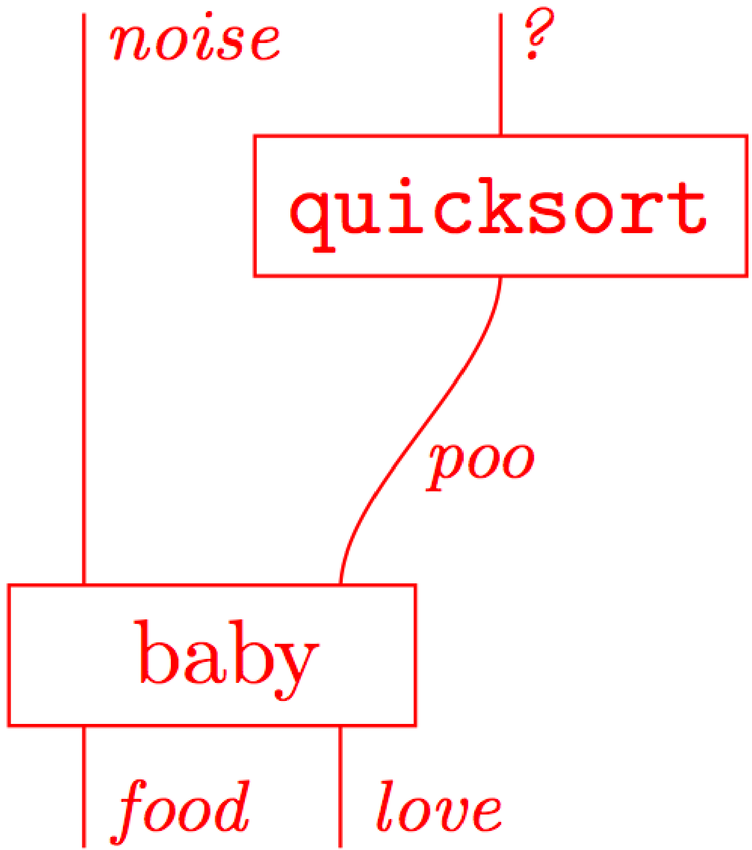


$:=$

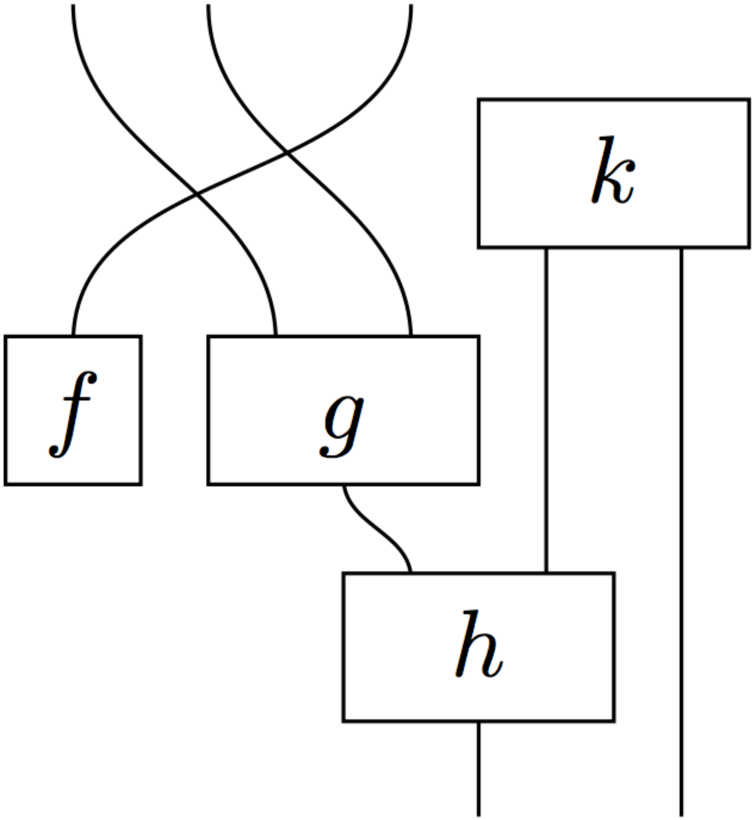


– composing processes –



– diagram equations –

– *tautologies* –



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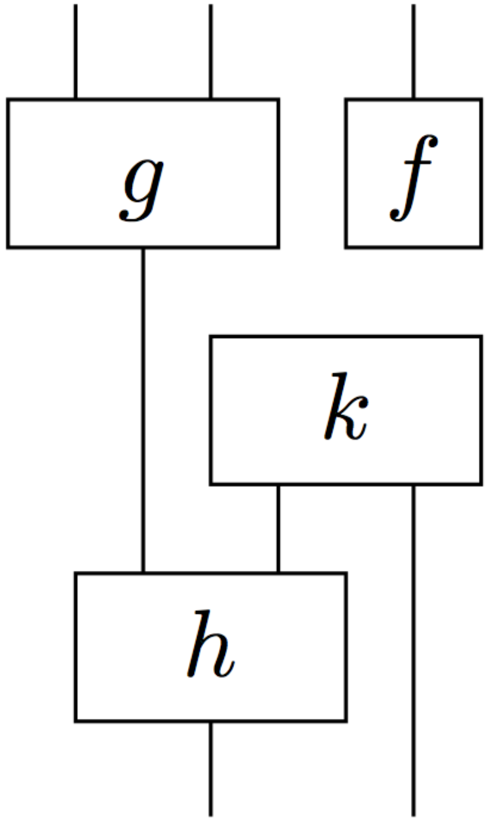

– process theories –

... consists of:

- **collection of systems**
- **collection of processes**
- **formalises ‘wiring together’**

... consists of:

- **collection of systems**
- **collection of processes**
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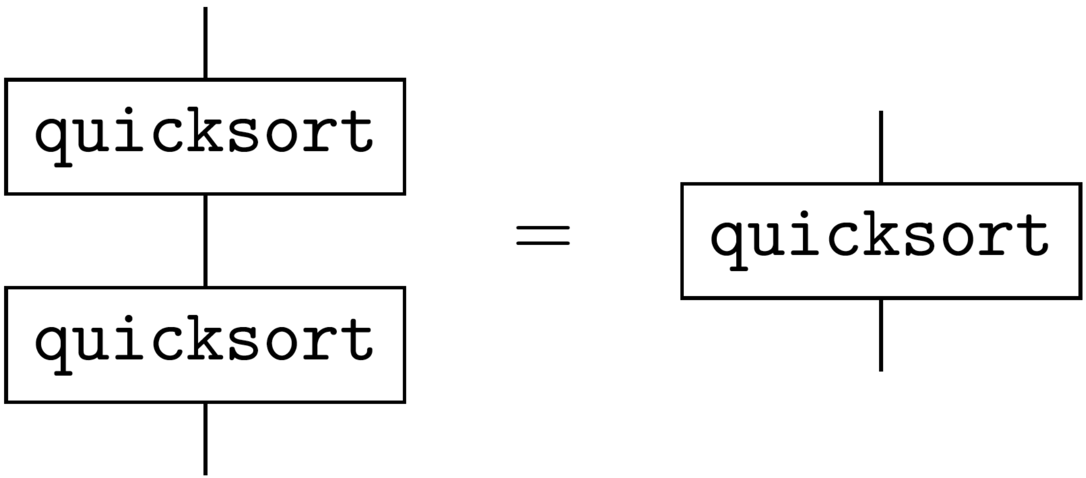
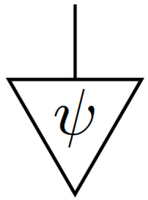
so in particular tells us:

- **when two diagrams are (non-freely) equal.**

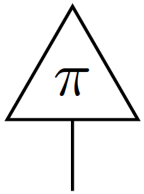


– *special processes/diagrams* –

State :=



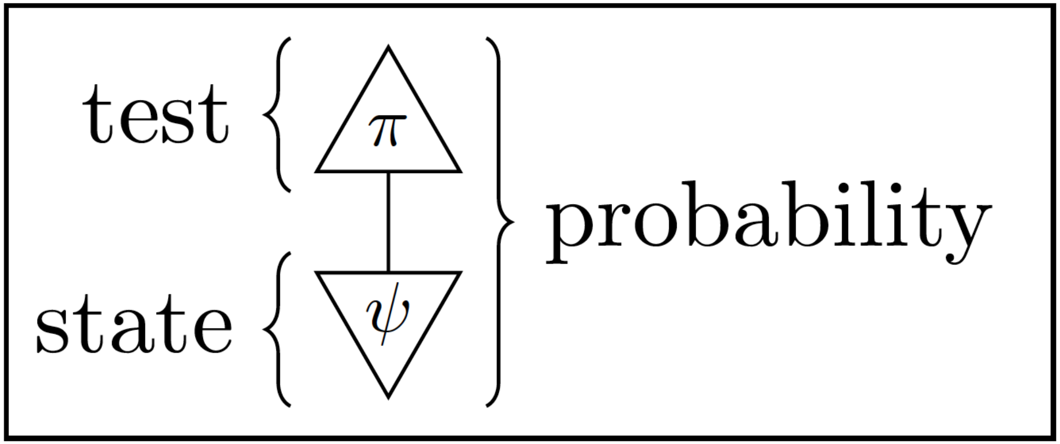
Effect / Test :=



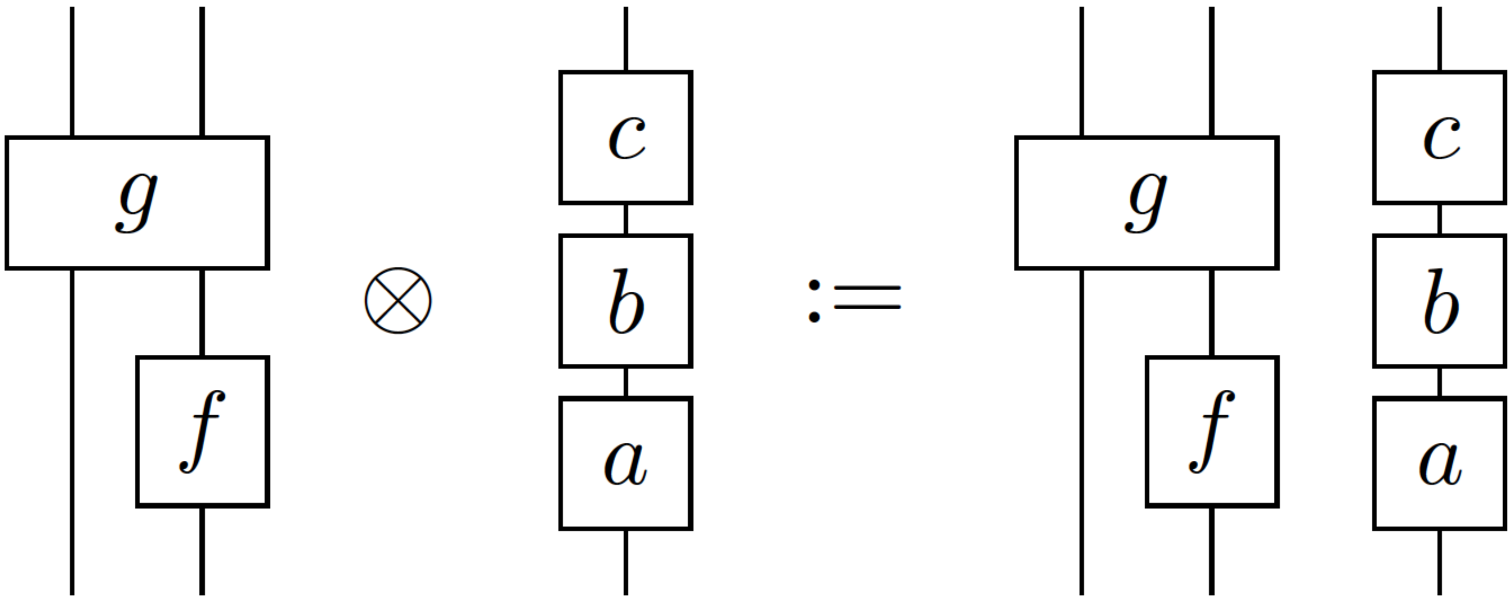
Number :=

– special processes/diagrams –

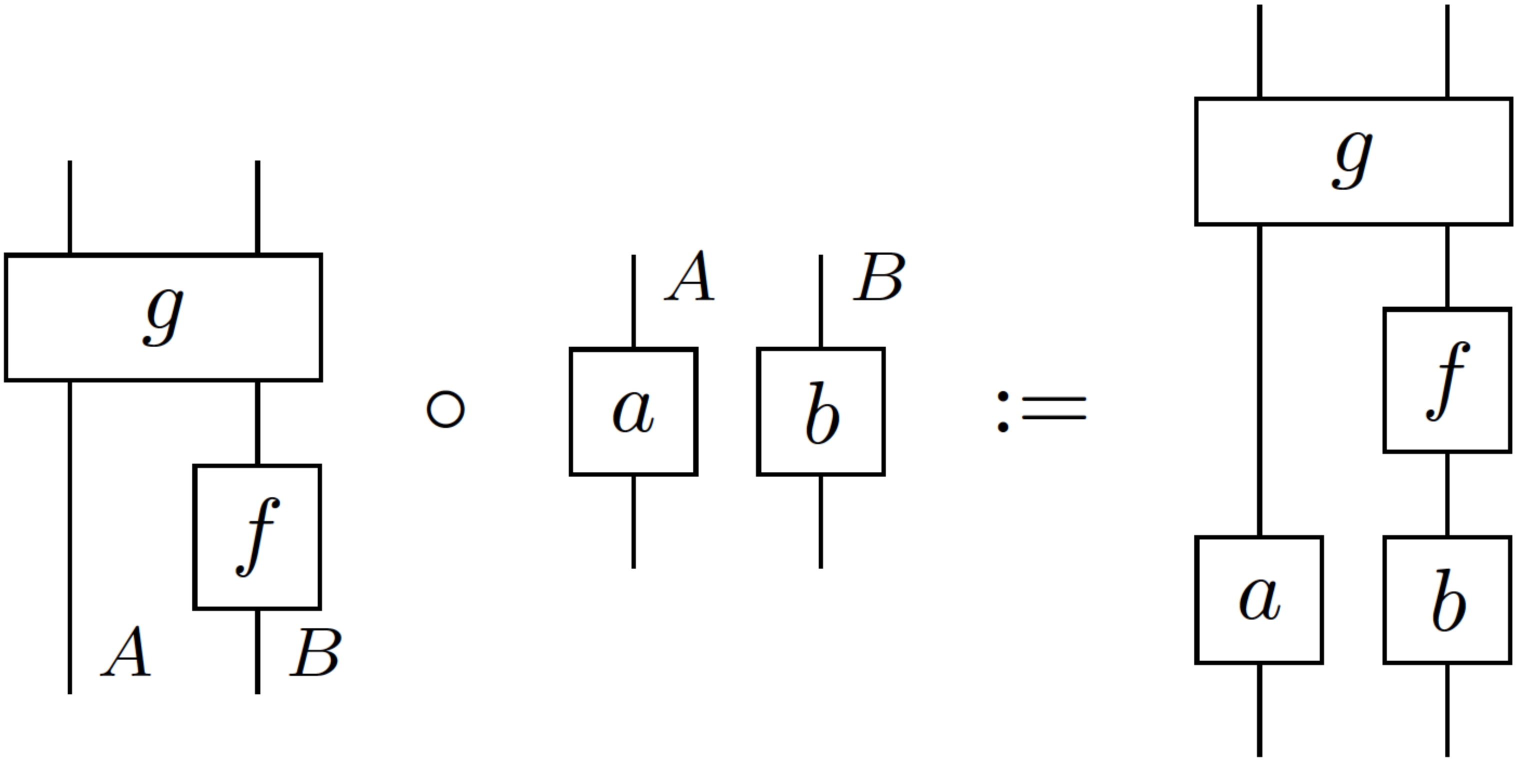
Born rule :=



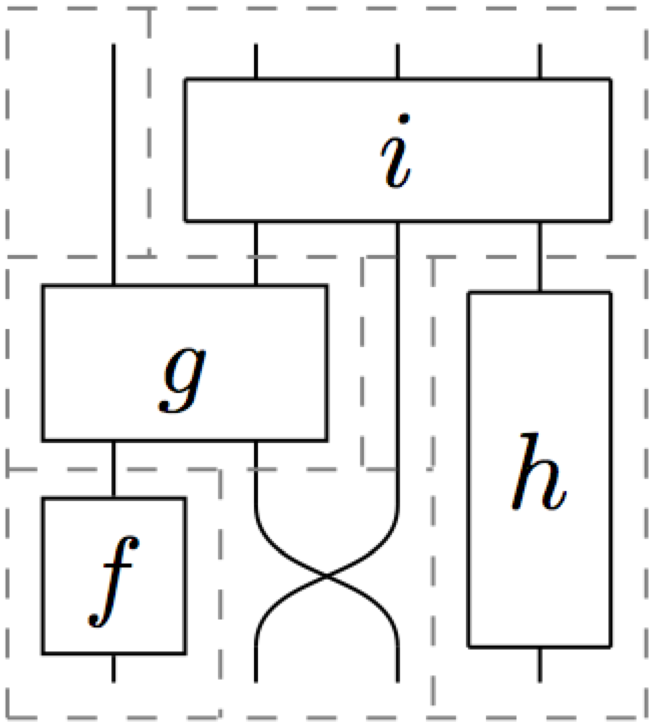
– "$f \otimes g$" := "f **while** g" –



– “$f \circ g$” := “f **after** g” –



Defn. ... := can be build with $\otimes$ and $\circ$.



Defn. ... := can be build with $\otimes$ and $\circ$.

Fact. ...are boring.

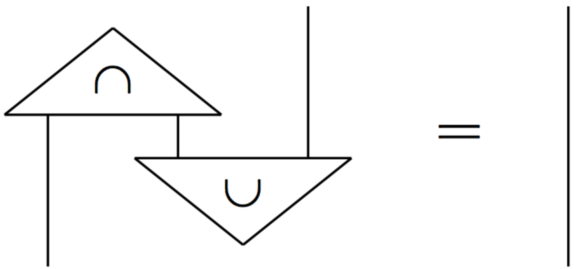
When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. ***I would not call that one but rather the characteristic trait of quantum mechanics****, the one that enforces its entire departure from classical lines of thought.*

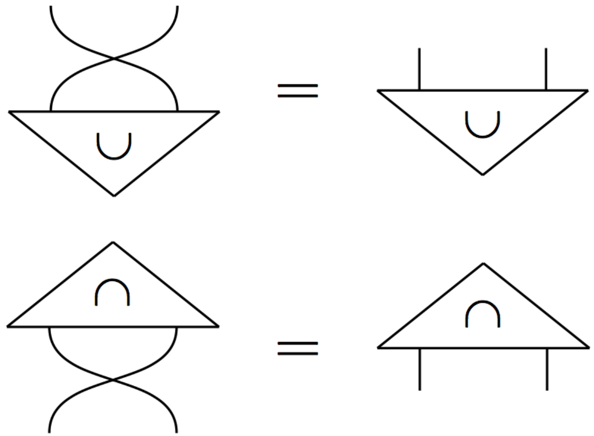
— Erwin Schrödinger, 1935.

– TFAE –

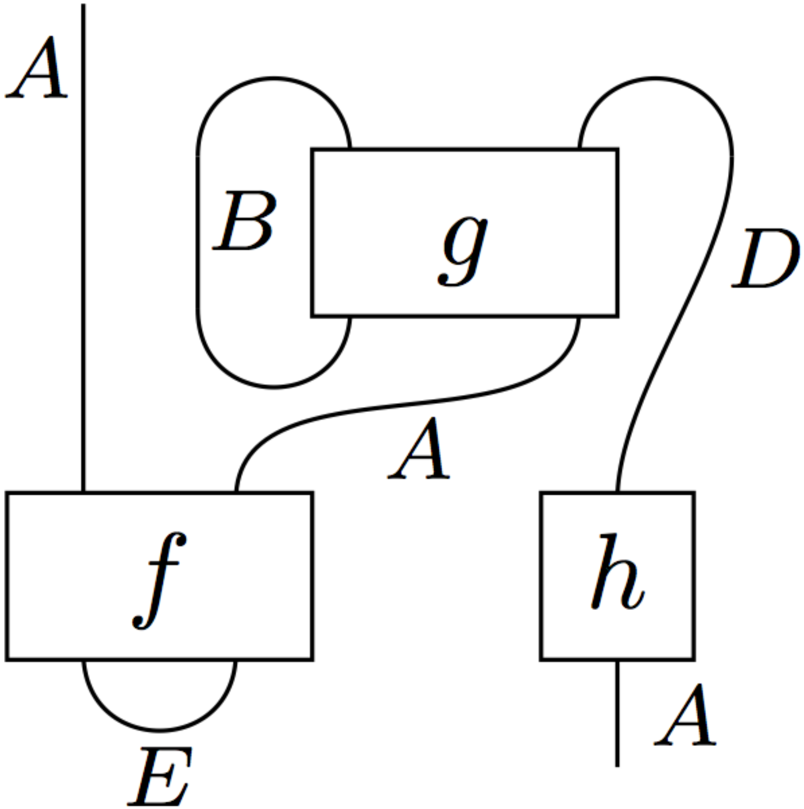
1. ‘Circuits’ with cup-state and cup-effect:

which satisfy:



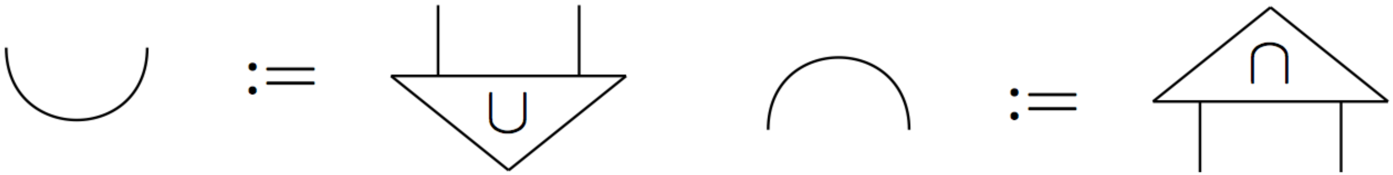


2. diagrams allowing in-in, out-out and out-in wiring:

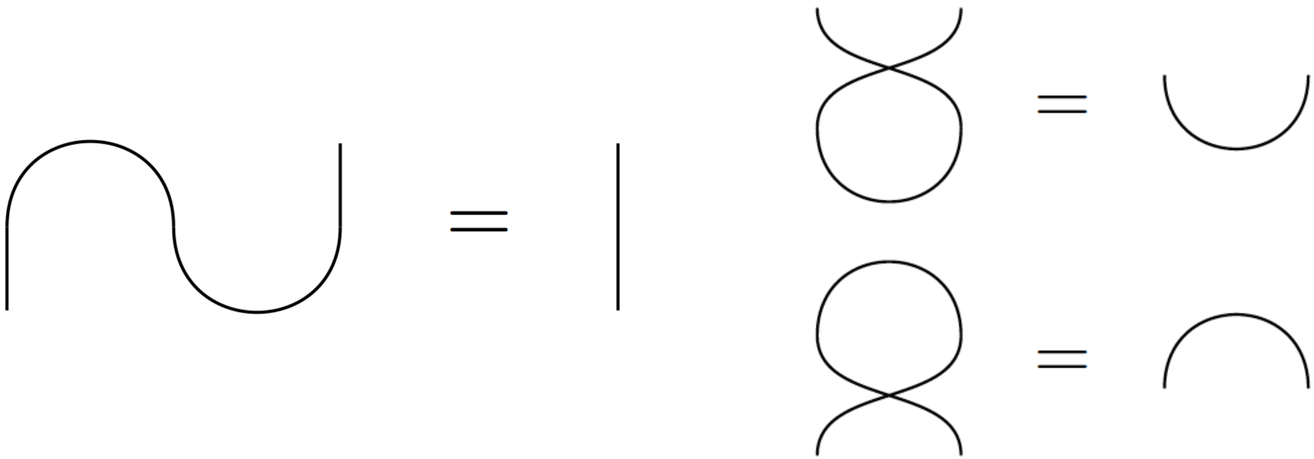


– TFAE –

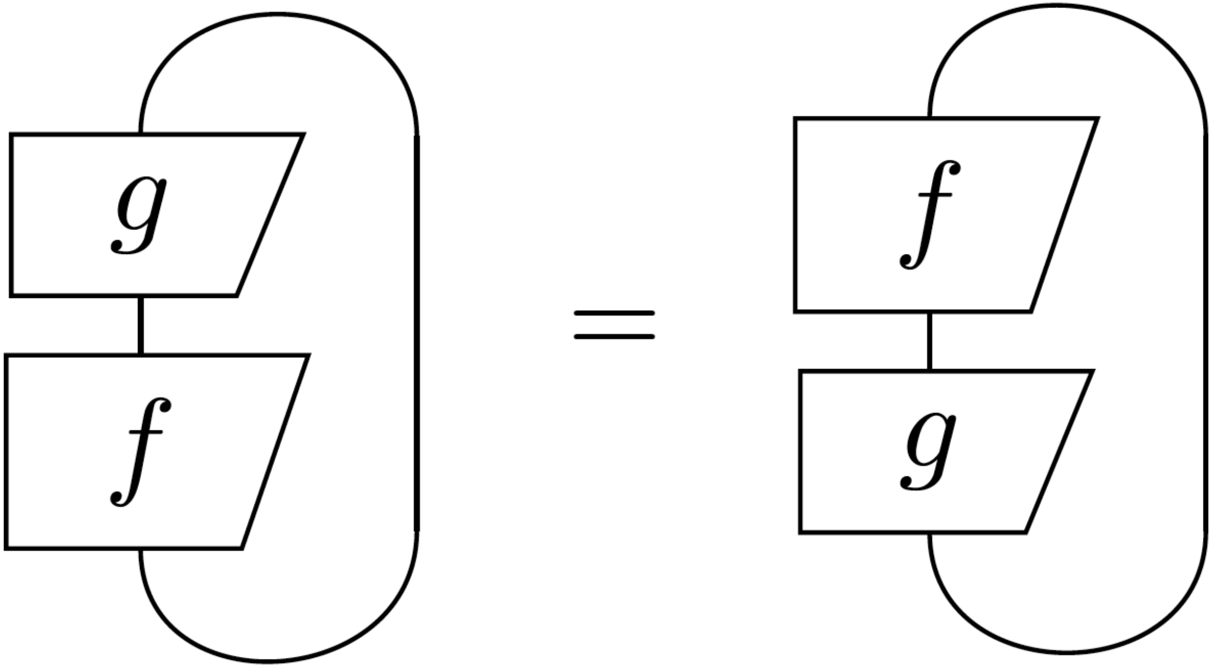
From **1.** to **2.**:



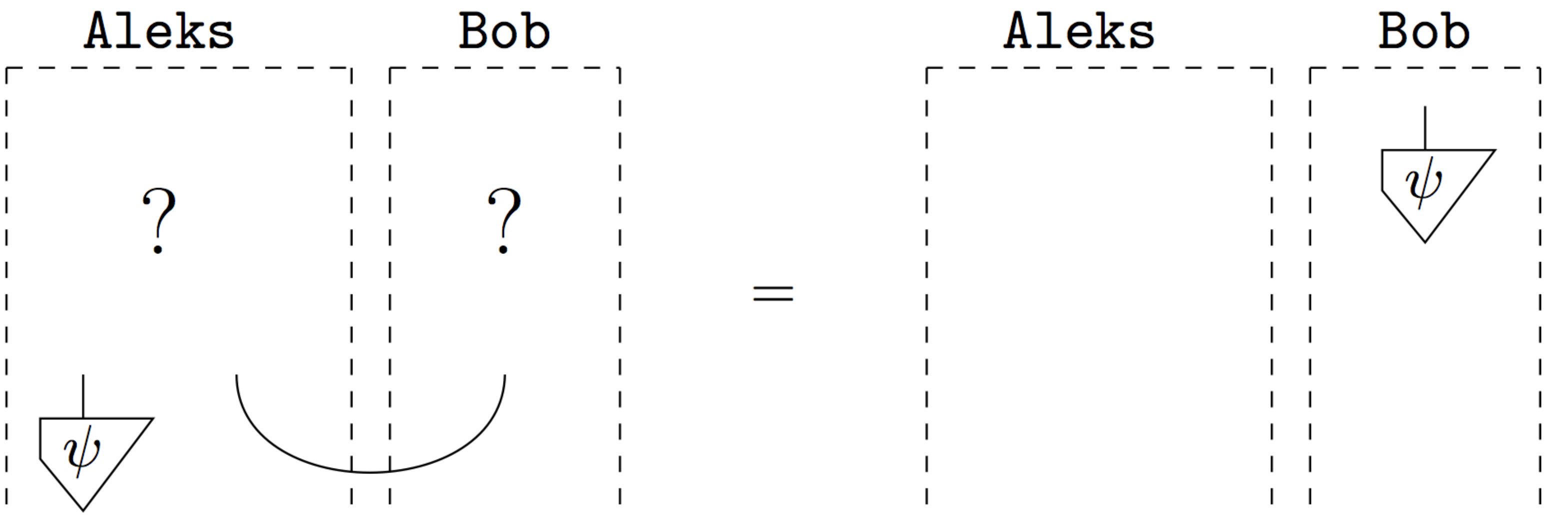
so that:



– tautology –



– *quantum teleportation* –



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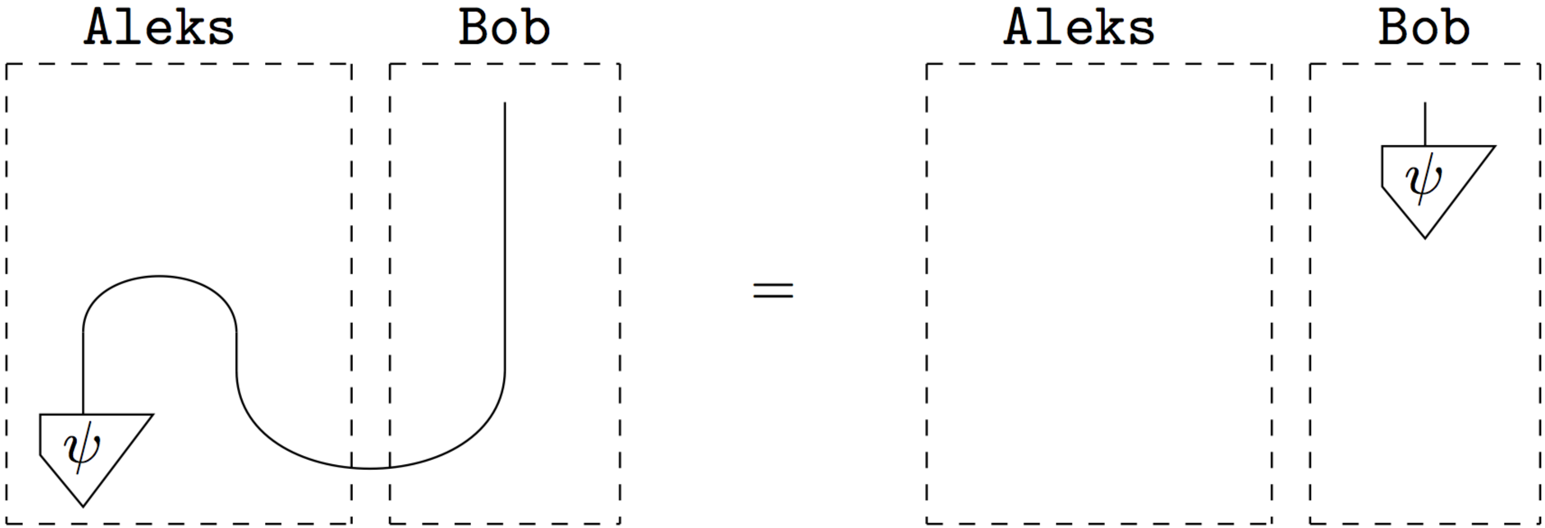
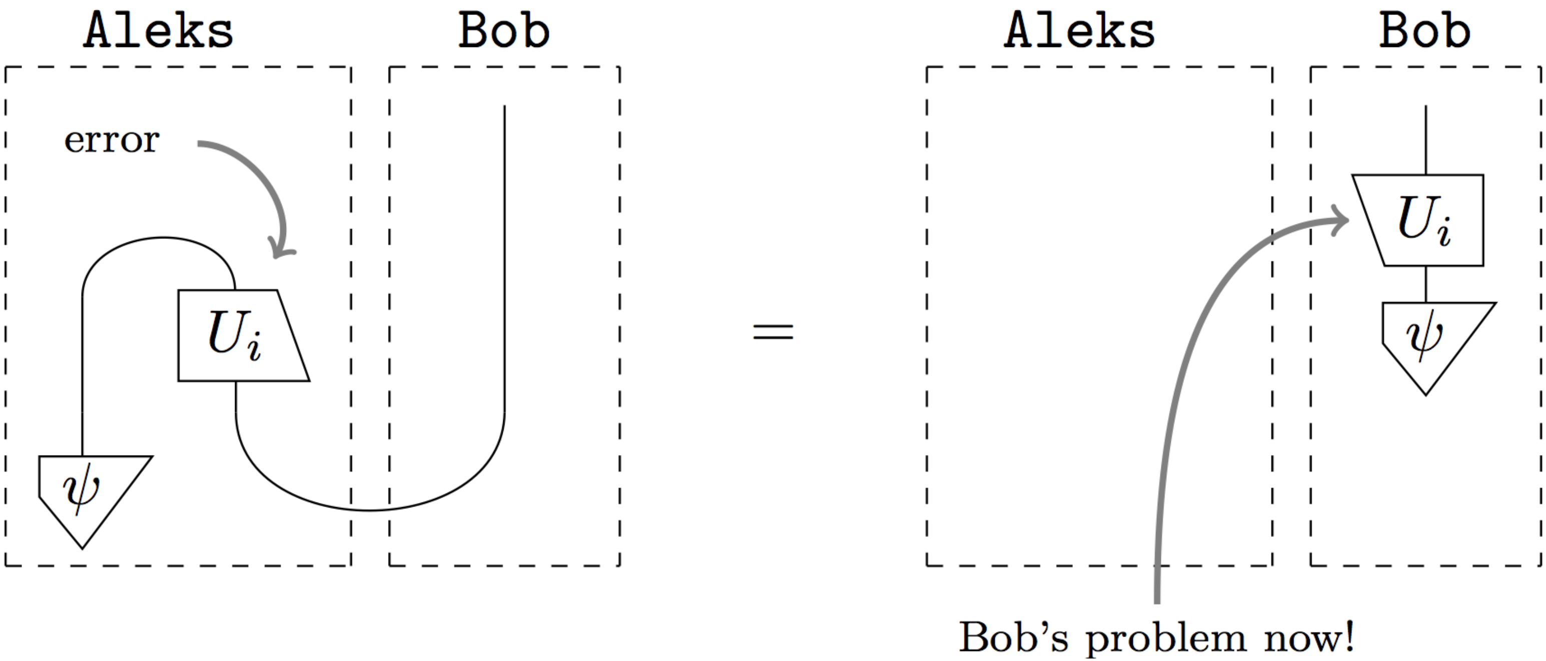


– quantum teleportation –

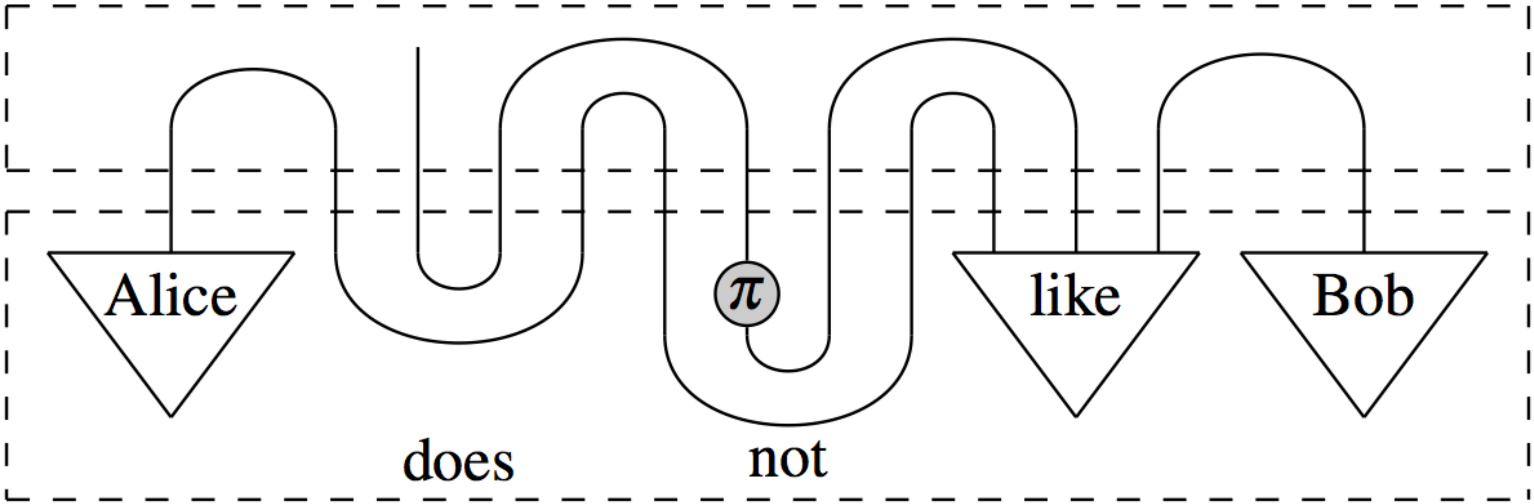


... what about natural language meaning?

... there are dictionaries for words ...

... why no dictionaries for sentences?

Computing the meaning of a sentence:



- Bottom part: meaning (e.g. vectors)
- Top part: grammar

B. Coecke, M. Sadrzadeh & S. Clark (2010) *Mathematical foundations for a compositional distributional model of meaning*. Lambek Festschrift. arXiv:1003.4394

Mathematics of grammar:

Lambek's Residuated monoids (1950's):

$$b \leq a \multimap c \Leftrightarrow a \cdot b \leq c \Leftrightarrow a \leq c \multimapinv b$$

so in particular,

$$a \cdot (a \multimap 1) \leq 1 \leq a \multimap (a \cdot 1)$$

$$(1 \multimapinv b) \cdot b \leq 1 \leq (1 \cdot b) \multimapinv b$$

Lambek's Pregroups (2000's):

$$a \cdot {}^{-1}a \leq 1 \leq {}^{-1}a \cdot a$$

$$b^{-1} \cdot b \leq 1 \leq b \cdot b^{-1}$$

Mathematics of grammar:

For noun type n, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

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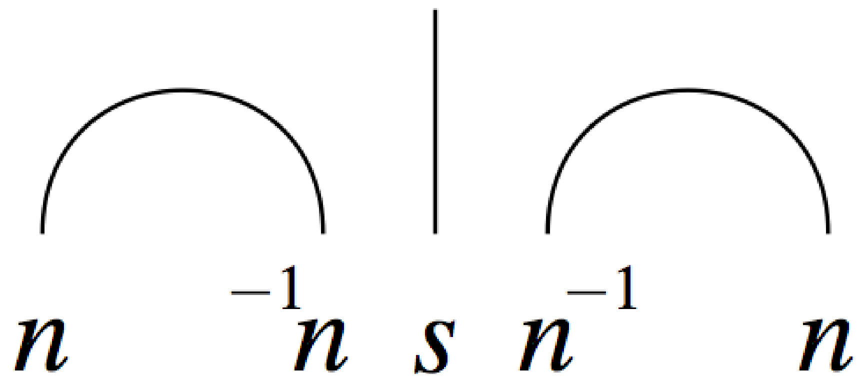
$$n \cdot {}^{-1}n \cdot s \cdot n^{-1} \cdot n \leq 1 \cdot s \cdot 1 \leq s$$

Mathematics of grammar:

For noun type n, verb type is ${}^{-1}n \cdot s \cdot n^{-1}$, so:

$$n \cdot {}^{-1}n \cdot s \cdot n^{-1} \cdot n \leq 1 \cdot s \cdot 1 \leq s$$

As a diagram:



Mathematics of grammar:

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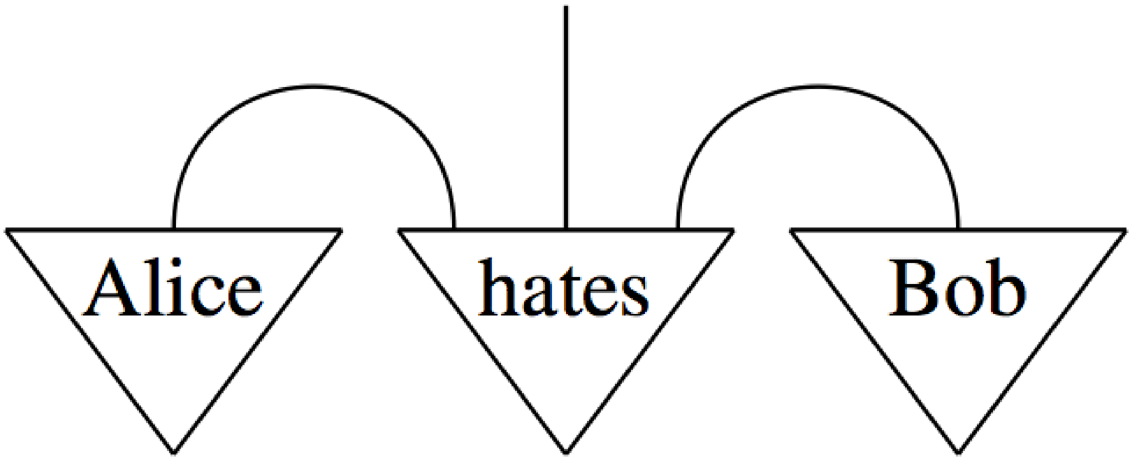


Algorithm for vector meaning composition:

1. Perform grammatical type reduction:

$$(word\ type\ 1)\ldots(word\ type\ n) \quad \rightsquigarrow \quad sentence\ type$$

2. Interpret diagrammatic type reduction as linear map:

$$f ::$$  $$\mapsto \left(\sum_i \langle ii|\right) \otimes \mathrm{id} \otimes \left(\sum_i \langle ii|\right)$$

3. Apply this map to tensor of word meaning vectors:

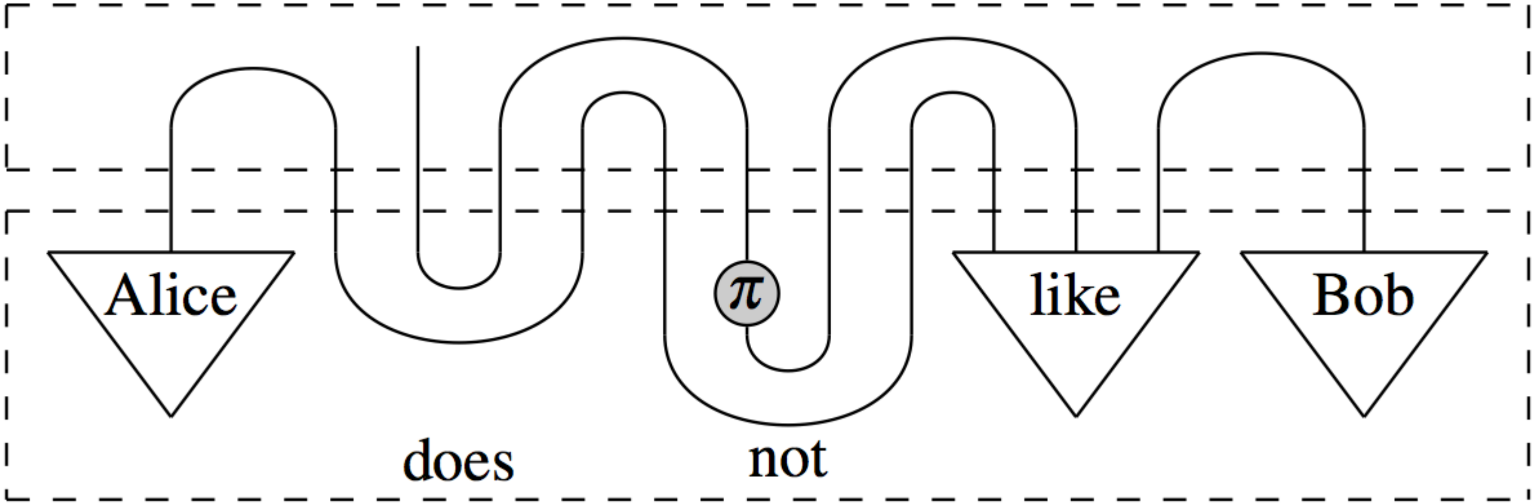
$$f\left(\overrightarrow{v}_1 \otimes \ldots \otimes \overrightarrow{v}_n\right)$$

Experimental evidence:

Model	ρ with cos	ρ with Eucl.
Verbs only	0.329	0.138
Additive	0.234	0.142
Multiplicative	0.095	0.024
Relational	0.400	0.149
Rank-1 approx. of relational	0.402	0.149
Separable	0.401	0.090
Copy-subject	0.379	0.115
Copy-object	0.381	0.094
Frobenius additive	**0.405**	0.125
Frobenius multiplicative	0.338	0.034
Frobenius tensored	**0.415**	0.010
Human agreement	0.60	

D. Kartsaklis & M. Sadrzadeh (2013) *Prior disambiguation of word tensors for constructing sentence vectors*. In EMNLP'13.

Logical meanings:



- Bottom part: meaning vectors
- Top part: grammar

B. Coecke, M. Sadrzadeh & S. Clark (2010) *Mathematical foundations for a compositional distributional model of meaning*. Lambek Festschrift. arXiv:1003.4394

Algorithm for NLP-meaning composition:

1. Perform grammatical type reduction:

$$(word\ type\ 1)\ldots(word\ type\ n) \ \rightsquigarrow \ sentence\ type$$

2. Interpret diagrammatic type reduction as NLP-map:

$$f :: $$  $$\mapsto \left(\sum_i \langle ii|\right) \otimes \mathrm{id} \otimes \left(\sum_i \langle ii|\right)$$

3. Apply this map to tensor of word NLP-states:

$$f\left(\overrightarrow{v}_1 \otimes \ldots \otimes \overrightarrow{v}_n\right)$$

Algorithm for XYZ-meaning composition:

1. Perform grammatical type reduction:

$$(word\ type\ 1)\ldots(word\ type\ n) \quad \rightsquigarrow \quad sentence\ type$$

2. Interpret diagrammatic type reduction as XYZ-map:

f ::  $\mapsto$ 'cap' $\otimes\, \mathrm{id} \otimes$ 'cap'

3. Apply this map to tensor of word XYZ-states:

$$f\,(v_1 \otimes \ldots \otimes v_n)$$

Examples:

1. Boolean matrices $\Rightarrow$ Montague

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1. Boolean matrices $\Rightarrow$ Montague

2. non-Boolean matrices $\Rightarrow$ logic dies

Examples:

1. Boolean matrices ⇒ Montague

2. non-Boolean matrices ⇒ logic dies

3. density matrices ⇒ ‘some’ logic re-emerges

- ambiguity

- lexical entailment

R. Piedeleu, D. Kartsaklis, B. Coecke & M. Sadrzadeh (2015) *Open system categorical quantum semantics in natural language processing*. CalCo. arXiv:1502.00831

D. Bankova, B. Coecke, M. Lewis & D. Marsden (2016) *Graded entailment for compositional distributional semantics*. arXiv:1601.04908

... what about meaning/cognition?

Algorithm for XYZ-meaning composition:

1. Perform grammatical type reduction:

$$(word\ type\ 1)\ldots(word\ type\ n) \quad \rightsquigarrow \quad sentence\ type$$

2. Interpret diagrammatic type reduction as XYZ-map:

$f ::$  $\mapsto$ 'cap' $\otimes \mathrm{id} \otimes$ 'cap'

3. Apply this map to tensor of word meaning XYZ-states:

$$f\,(v_1 \otimes \ldots \otimes v_n)$$

Algorithm for cog.-meaning composition:

1. Perform grammatical type reduction:

$$(word\ type\ 1)\ldots(word\ type\ n) \quad \rightsquigarrow \quad sentence\ type$$

2. Interpret diagrammatic type reduction as cog.-map:

$f ::$  $\mapsto$ ‘cap’ $\otimes\, \mathrm{id} \otimes$ ‘cap’

3. Apply this map to tensor of word meaning cog.-states:

$$f\,(v_1 \otimes \ldots \otimes v_n)$$

General recipe:

1. Pick compositional mechanism **CM** (e.g. grammar)

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2. Organise meaning/concept/cognitive spaces & maps in tensor-category $\otimes$-**Cat** that matches **CM**.

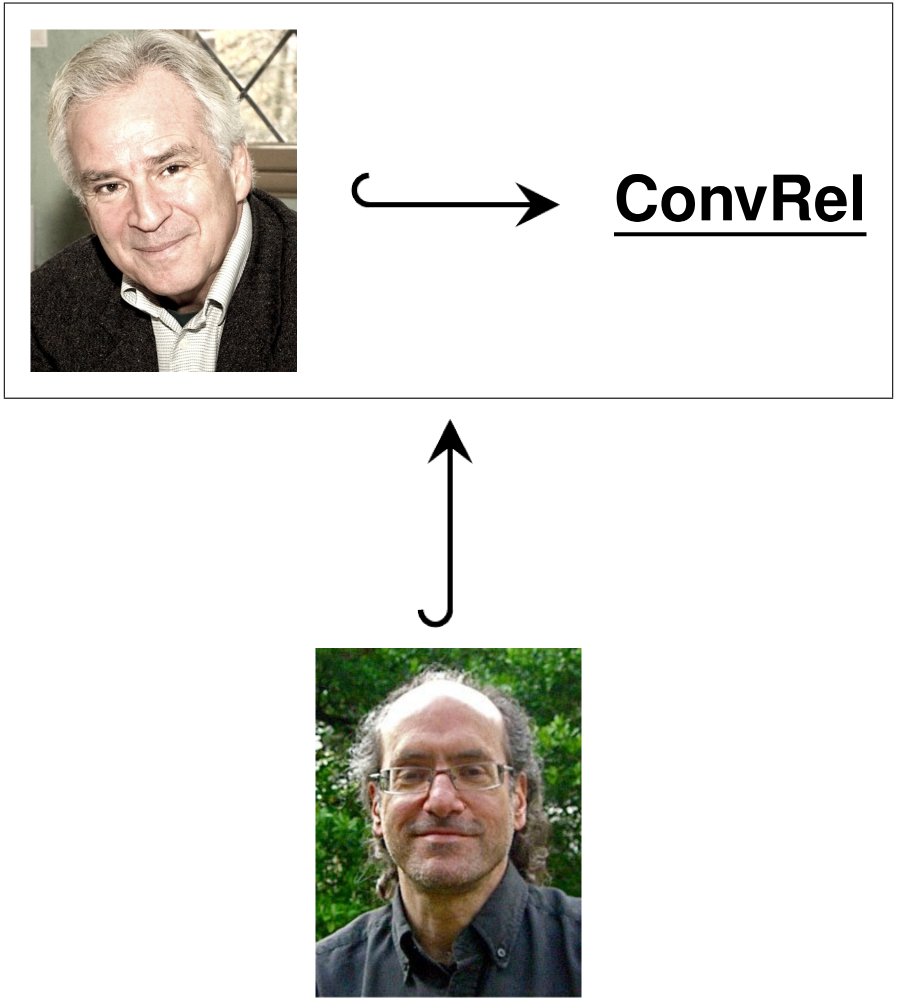
General recipe:

1. Pick compositional mechanism $\mathbf{CM}$ (e.g. grammar)

2. Organise meaning/concept/cognitive spaces & maps in tensor-category $\otimes$-$\mathbf{Cat}$ that matches $\mathbf{CM}$.

3. Carry over compositional reasoning:

$$\mathbf{CM} \longrightarrow \otimes\text{-}\mathbf{Cat}$$



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Y. Al-Mehairi, B. Coecke & M. Lewis (2016) *Compositional Distributional Cognition*. QI'16.

A **convex algebra** is set A and 'mixing' function:

$$\alpha : D(A) \to A$$

i.e.:

$$\alpha(|a\rangle) = a$$

$$\alpha\left(\sum\nolimits_{i,j} p_i q_{i,j} |a_{i,j}\rangle\right) = \alpha\left(\sum\nolimits_{i} p_i |\alpha(\sum\nolimits_{j} q_{i,j} |a_{i,j}\rangle)\rangle\right)$$

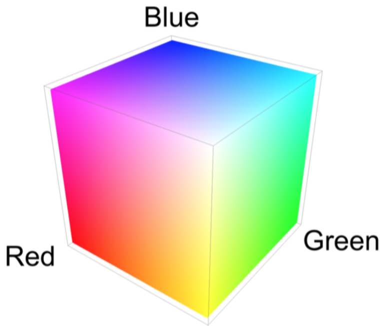
A **convex relation** of type $(A, \alpha) \to (B, \beta)$ is relation:

$$R : A \to B$$

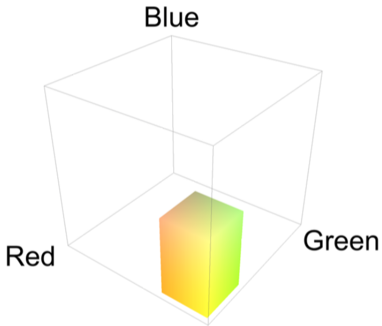
that 'commutes with mixtures':

$$(\forall i. R(a_i, b_i)) \Rightarrow R\left(\sum_i p_i a_i, \sum_i p_i b_i\right)$$

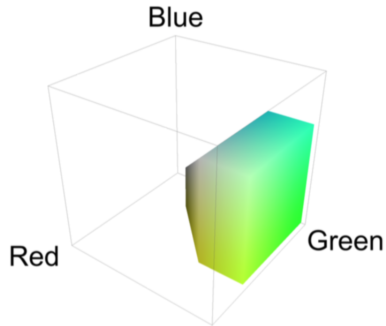
$$N_{food} = N_{colour} \otimes N_{taste} \otimes N_{texture}$$



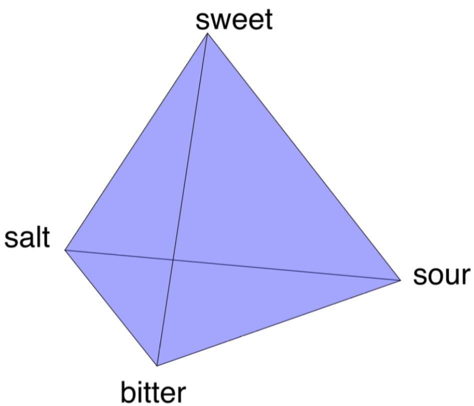
(a) The RGB colour cube



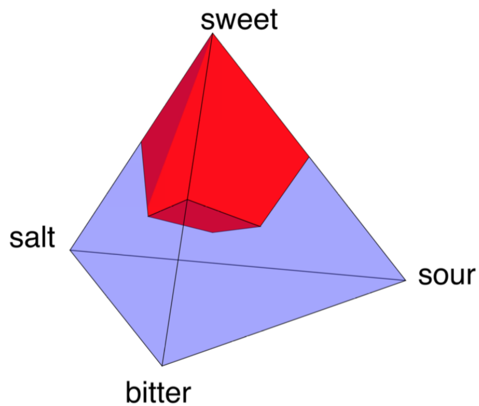
(b) Property p_{yellow}



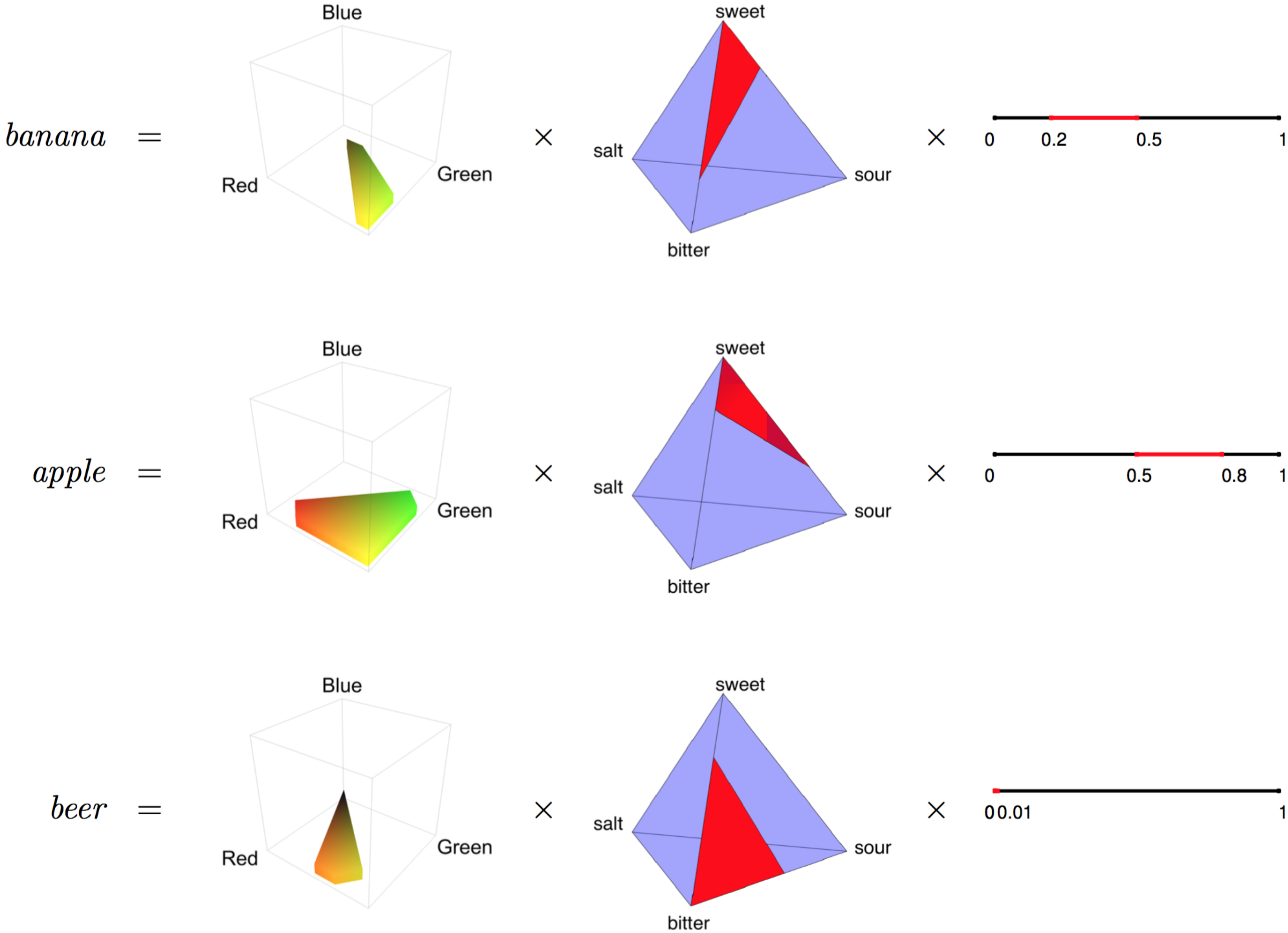
(c) Property p_{green}



(a) The taste tetrahedron

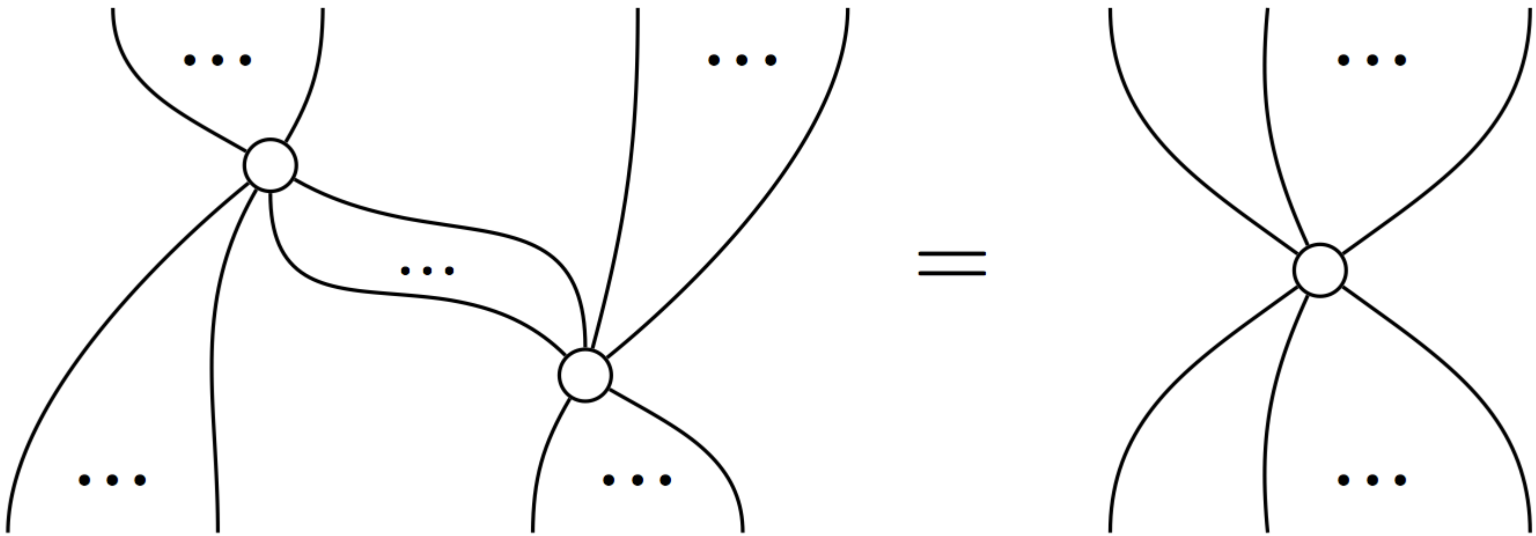


(b) The property p_{sweet}



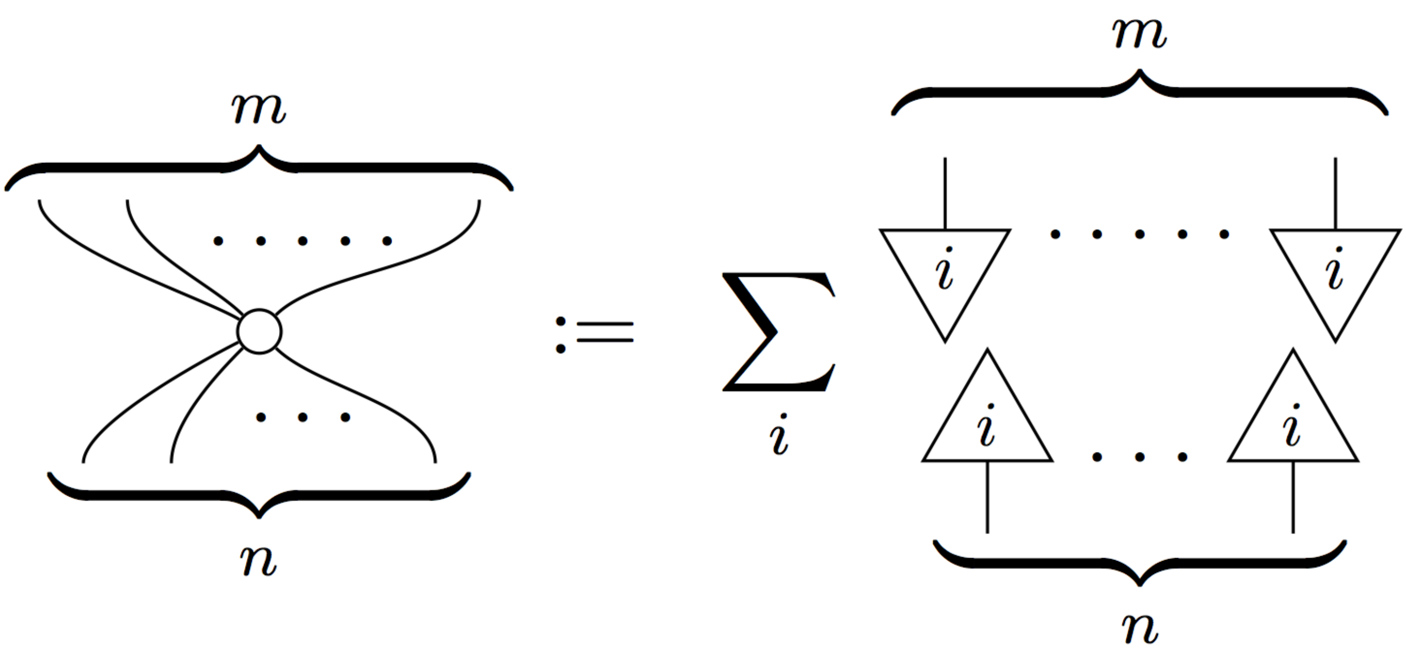
banana =
Blue
Red
Green
×
sweet
salt
sour
bitter
×
0
0.2
0.5
1
apple =
Blue
Red
Green
×
sweet
salt
sour
bitter
×
0
0.5
0.8
1
beer =
Blue
Red
Green
×
sweet
salt
sour
bitter
×
0
0.01
1

– classicality as spiders –



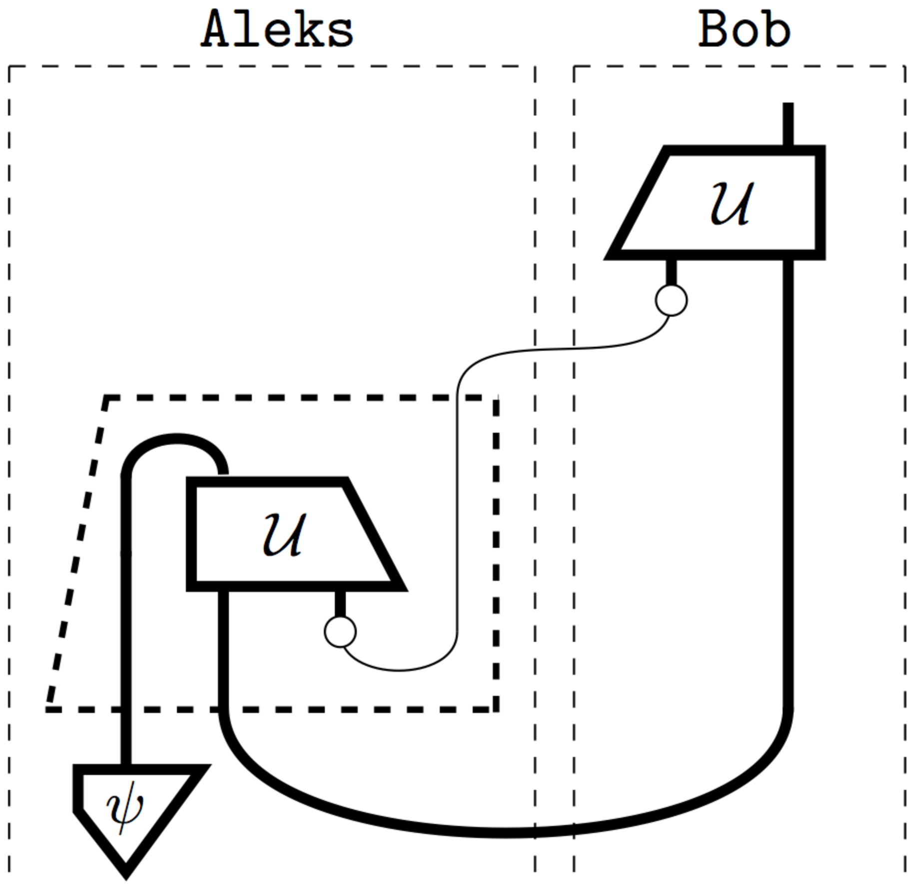
B. Coecke, É. O. Paquette and D. Pavlović (2010) Classical and quantum structuralism. CUP-book. arXiv:0904.1997.

– *classicality as spiders* –

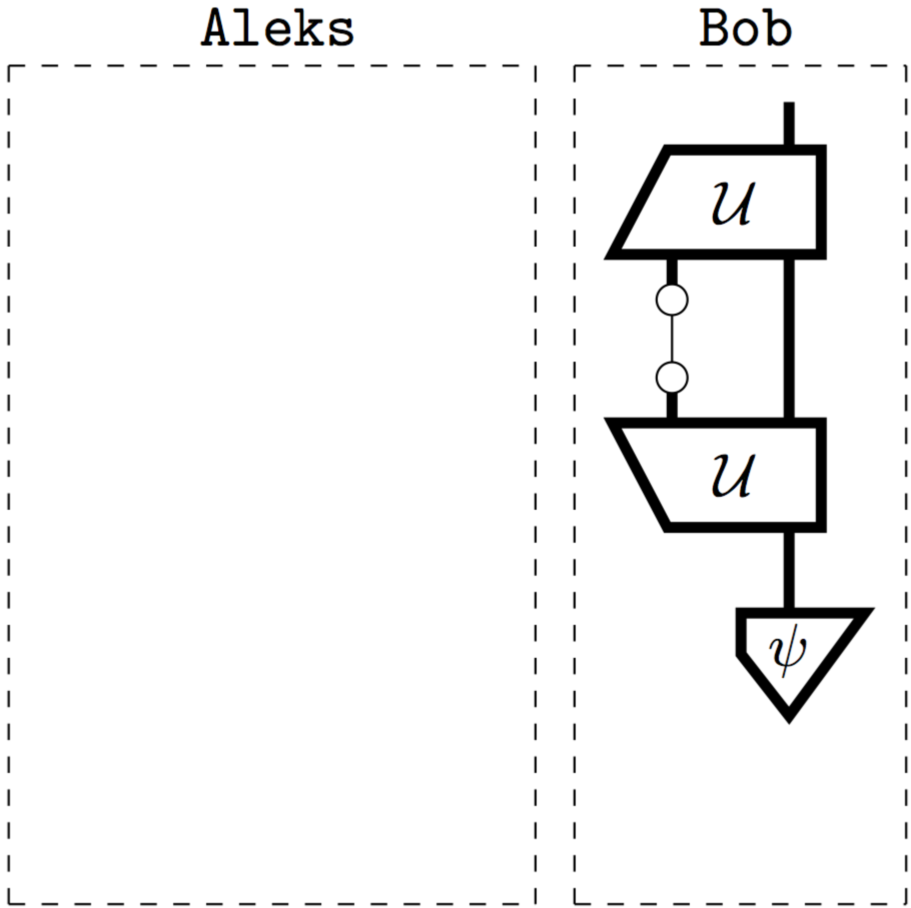


B. Coecke, D. Pavlovic and J. Vicary (2013) *A new description of orthogonal bases*. MSCS. arXiv:0810.0812

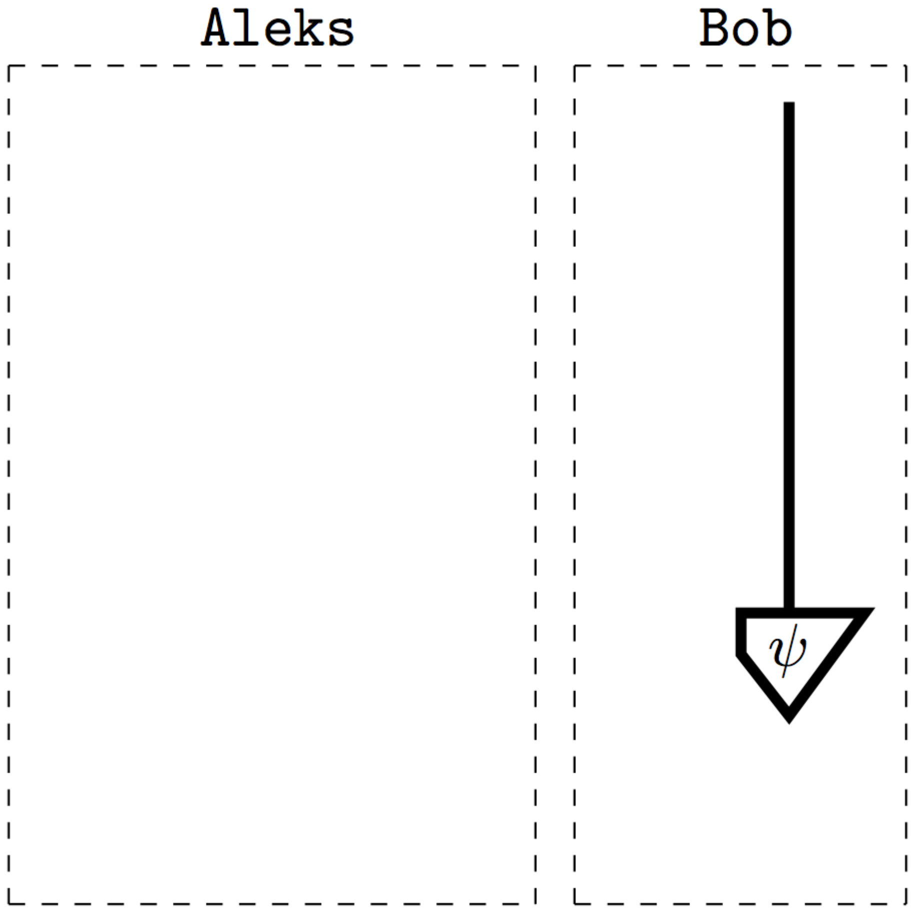
– teleportation diagrammatically –



– teleportation diagrammatically –

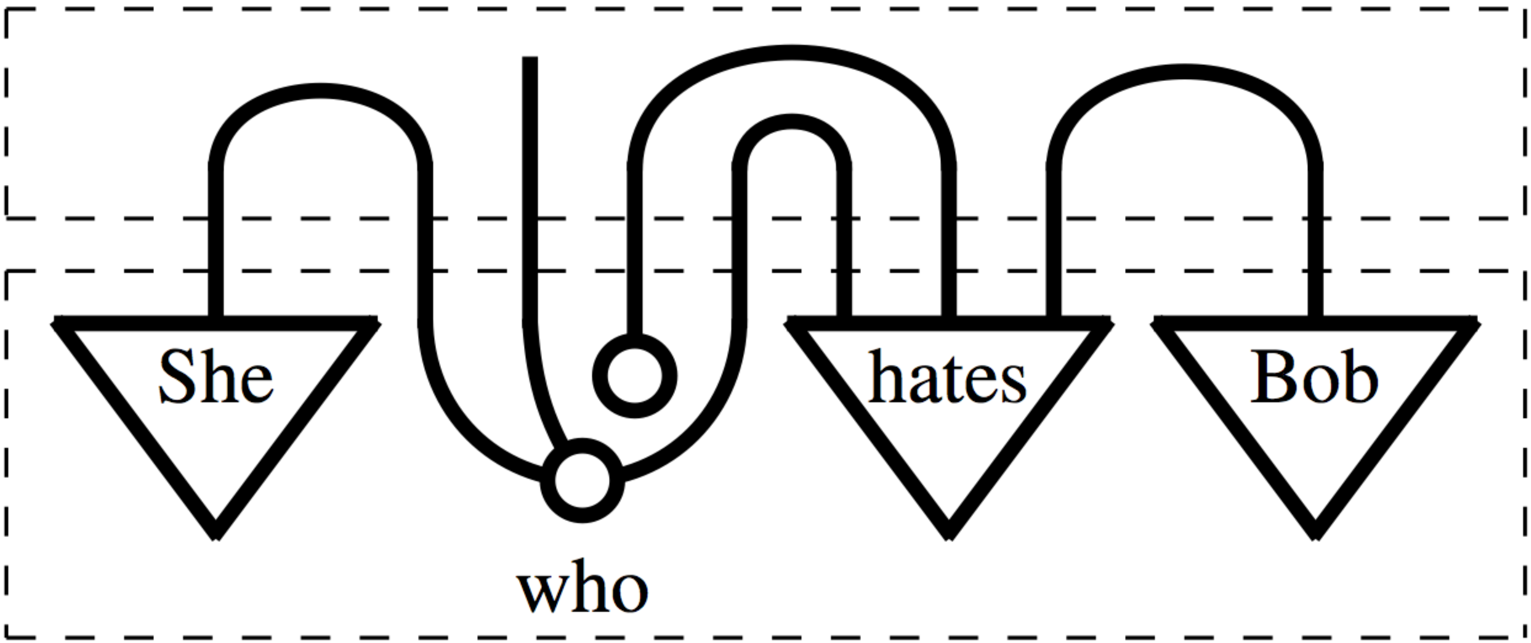


– teleportation diagrammatically –



... what about meaning/cognition?

Relative pronouns:



M. Sadrzadeh, B. Coecke & S. Clark (2013–2014) *The Frobenius anatomy of word meaning I & II*. Journal of Logic and Computation. arXiv:1404.5278

$$\rho_{she} := \sum \begin{cases} |Alice\rangle\langle Alice| \\ |Beth\rangle\langle Beth| \\ \ldots \end{cases}$$

$$\rho_{hates} := \sum \begin{cases} |Alice\rangle\langle Alice| \otimes \rho' \otimes |Bob\rangle\langle Bob| \\ |Beth\rangle\langle Beth| \otimes \rho'' \otimes |Colin\rangle\langle Colin| \\ \ldots \end{cases}$$

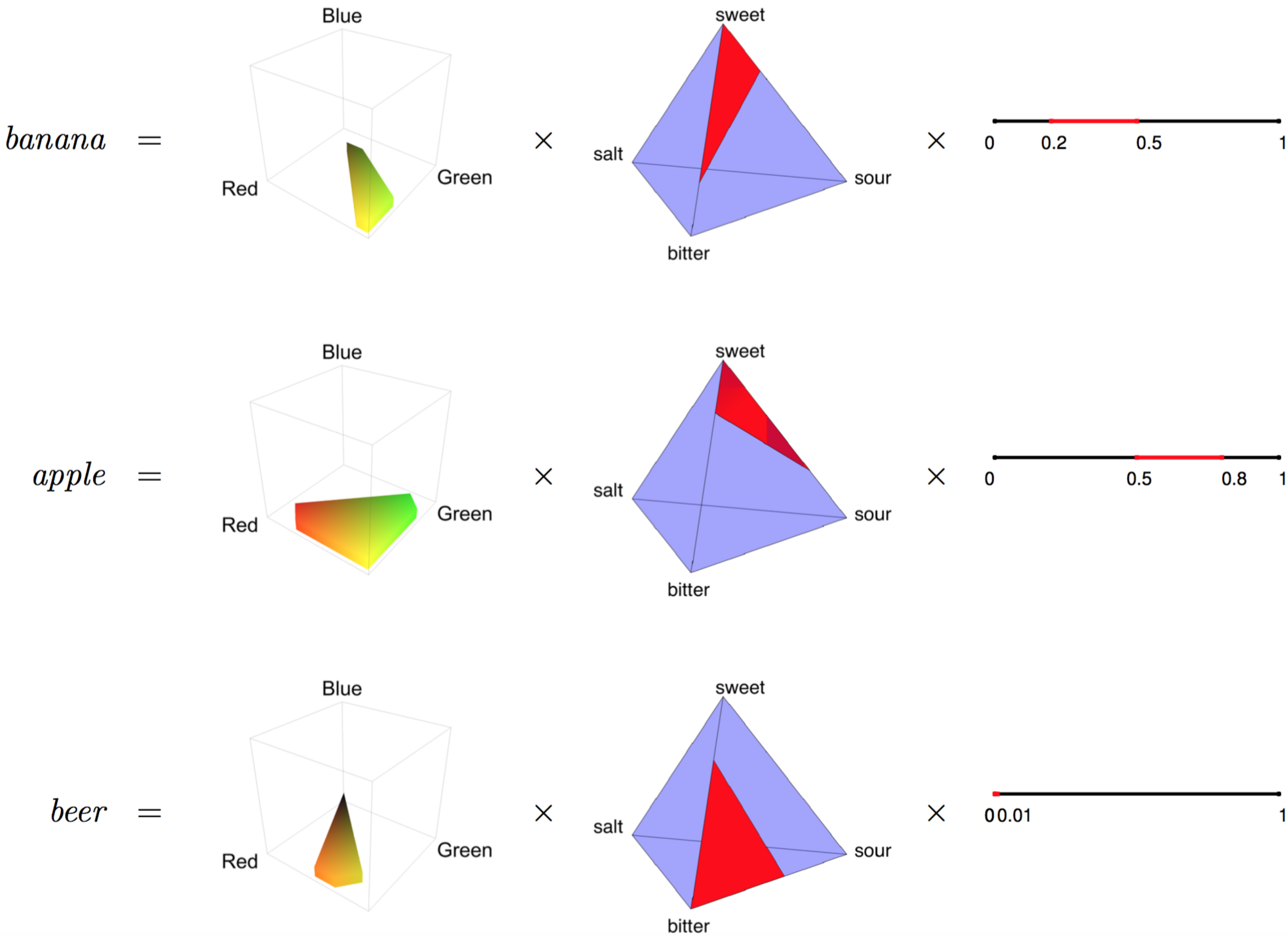
$$\rho_{Bob} := |Bob\rangle\langle Bob|$$

$$\rho_{she} := \sum \begin{cases} |Alice\rangle\langle Alice| \\ |Beth\rangle\langle Beth| \\ \ldots \end{cases}$$

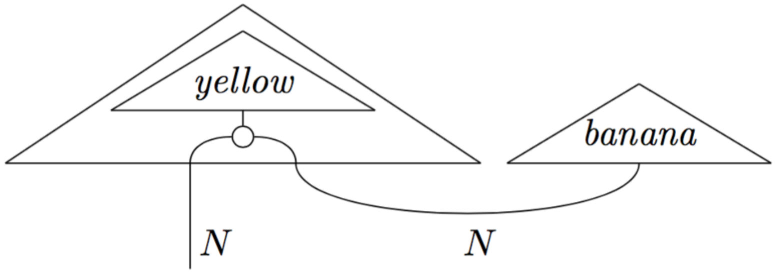
$$\rho_{hates} := \sum \begin{cases} |Alice\rangle\langle Alice| \otimes \rho' \otimes |Bob\rangle\langle Bob| \\ |Beth\rangle\langle Beth| \otimes \rho'' \otimes |Colin\rangle\langle Colin| \\ \ldots \end{cases}$$

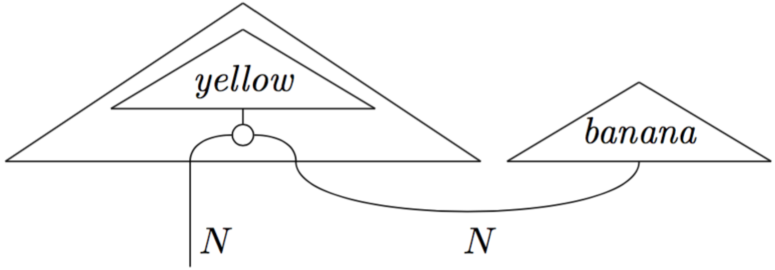
$$\rho_{Bob} := |Bob\rangle\langle Bob|$$

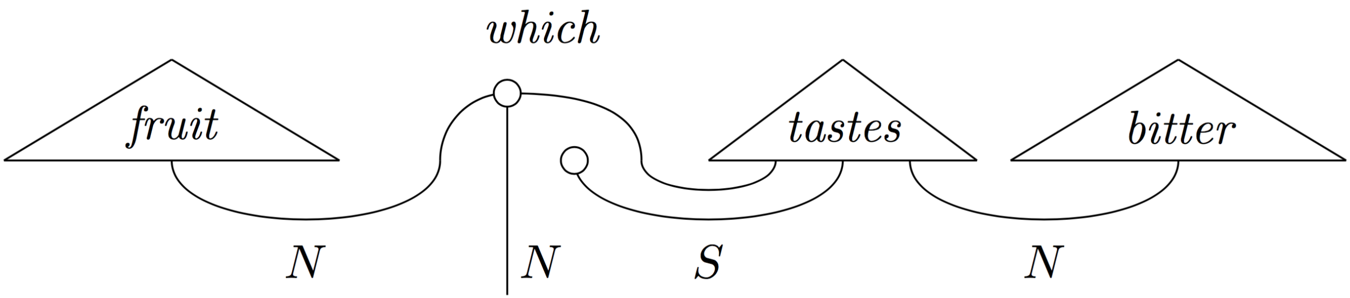
$$\rho_{sentence} := |Alice\rangle\langle Alice|$$

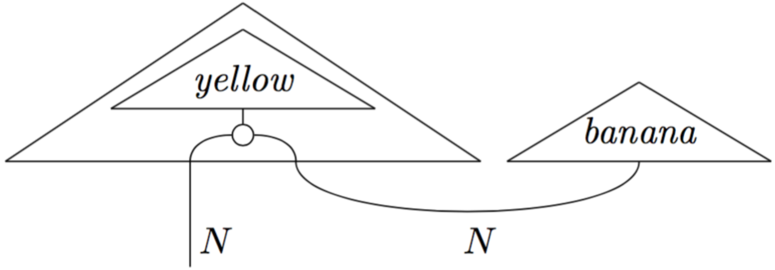


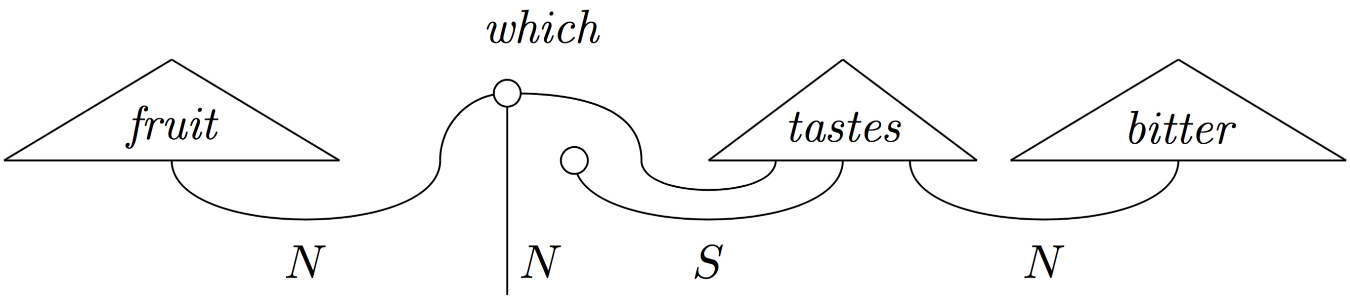
banana =
Blue
Red
Green
×
sweet
salt
sour
bitter
×
0
0.2
0.5
1
apple =
Blue
Red
Green
×
sweet
salt
sour
bitter
×
0
0.5
0.8
1
beer =
Blue
Red
Green
×
sweet
salt
sour
bitter
×
0 0.01
1


yellow
banana
N
N


yellow
banana
N
N


which
fruit
tastes
bitter
N
N
S
N





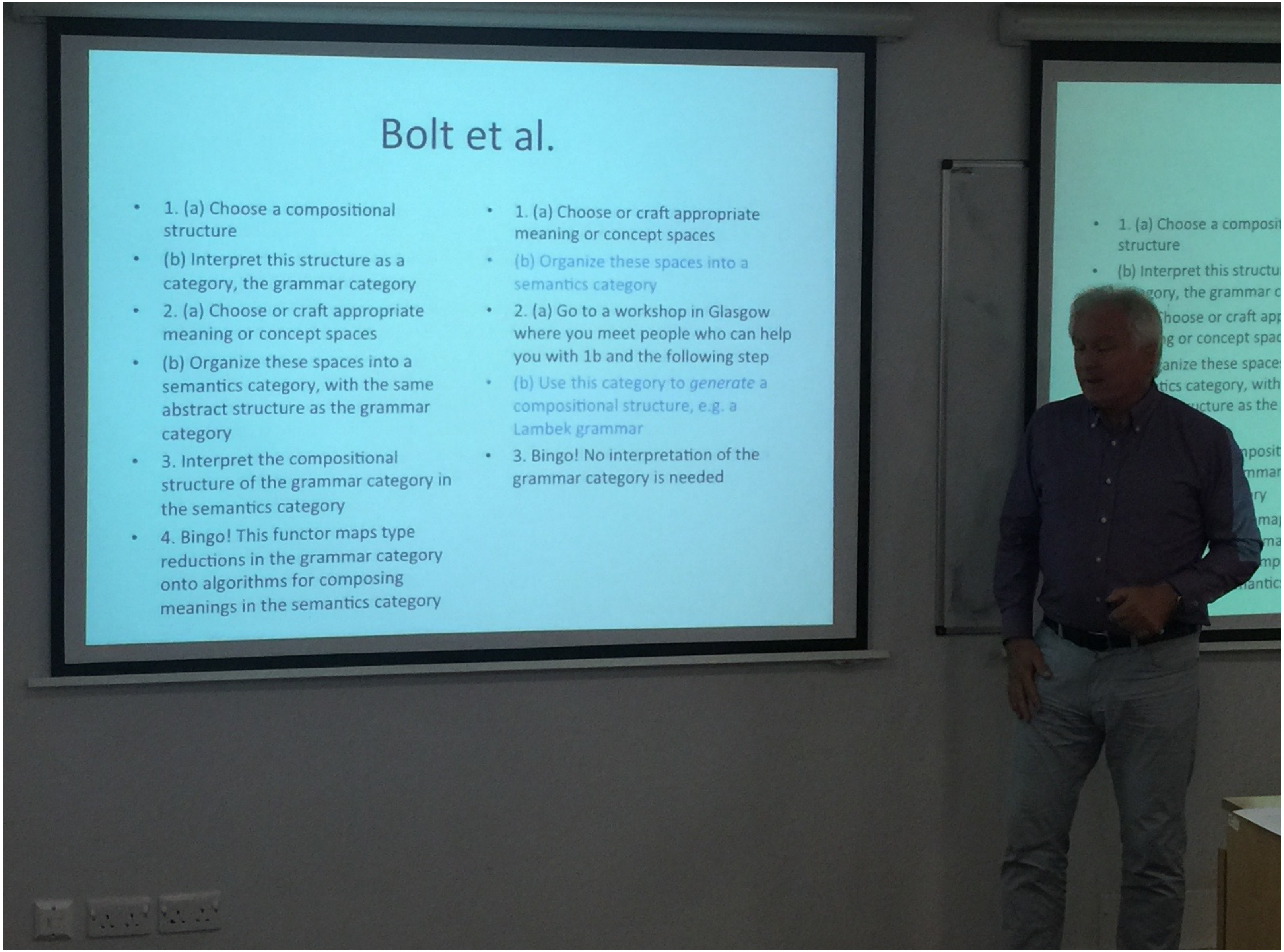
Fruit which tastes bitter

$= (\mu_N \times \iota_S \times \epsilon_N)(Conv(\textit{bananas} \cup \textit{apples}) \times \textit{taste} \times \textit{bitter})$

$= (\mu_N \times \iota_S)(Conv(\textit{bananas} \cup \textit{apples}) \times (\textit{green banana} \times \{(0,0)\}))$

$= \mu_N(Conv(\textit{bananas} \cup \textit{apples}) \times (\textit{green banana}))$

$= \textit{green banana}$


Bolt et al.
1. (a) Choose a compositional structure
(b) Interpret this structure as a category, the grammar category
2. (a) Choose or craft appropriate meaning or concept spaces
(b) Organize these spaces into a semantics category, with the same abstract structure as the grammar category
3. Interpret the compositional structure of the grammar category in the semantics category
4. Bingo! This functor maps type reductions in the grammar category onto algorithms for composing meanings in the semantics category
1. (a) Choose or craft appropriate meaning or concept spaces
(b) Organize these spaces into a semantics category
2. (a) Go to a workshop in Glasgow where you meet people who can help you with 1b and the following step
(b) Use this category to *generate* a compositional structure, e.g. a Lambek grammar
3. Bingo! No interpretation of the grammar category is needed

Concrete ongoing projects:

- Entanglement & superselection for meaning $\otimes$ grammar.

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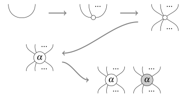
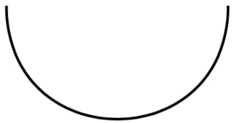
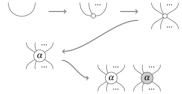
- Entanglement & superselection for meaning $\otimes$ grammar.

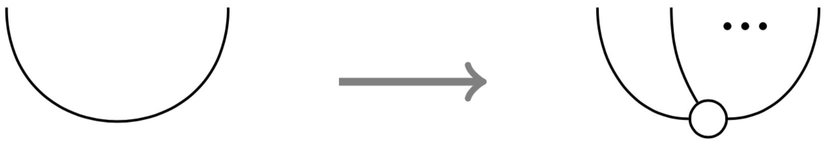
- Sentences as movies (i.e. 3+1D space)meaning model.

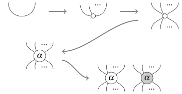
— structural evolution  —

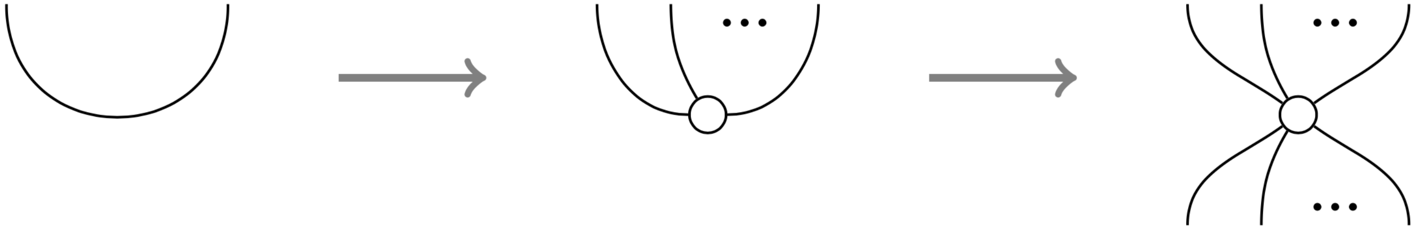


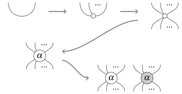
— structural evolution —

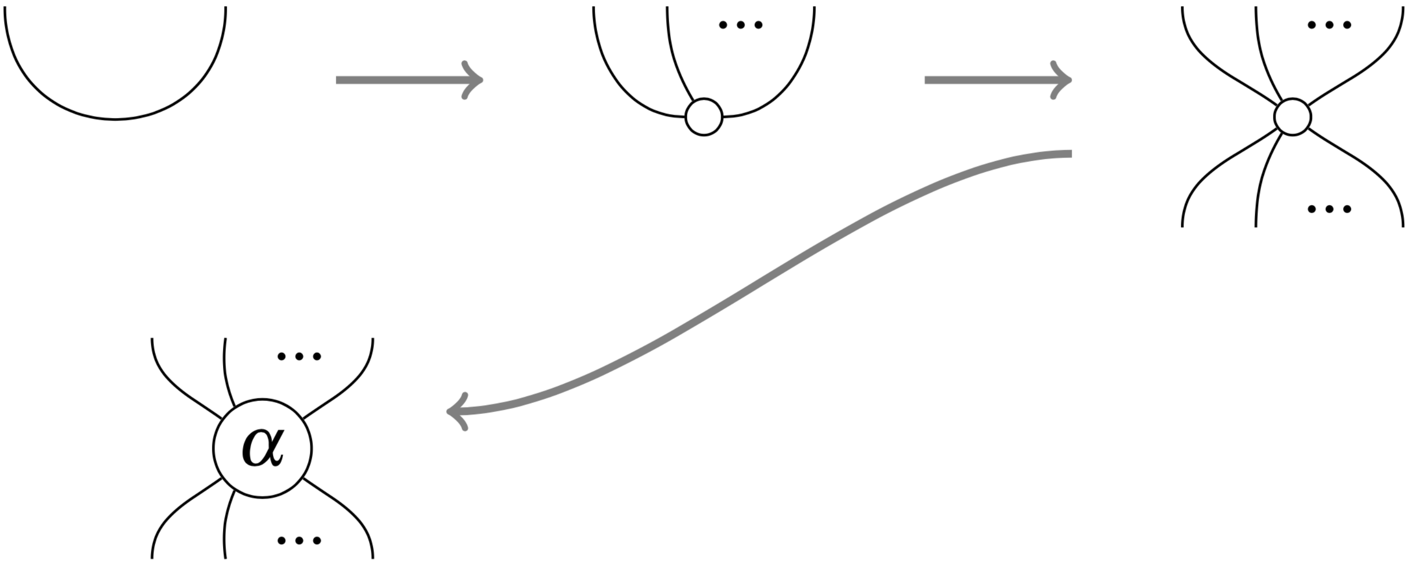




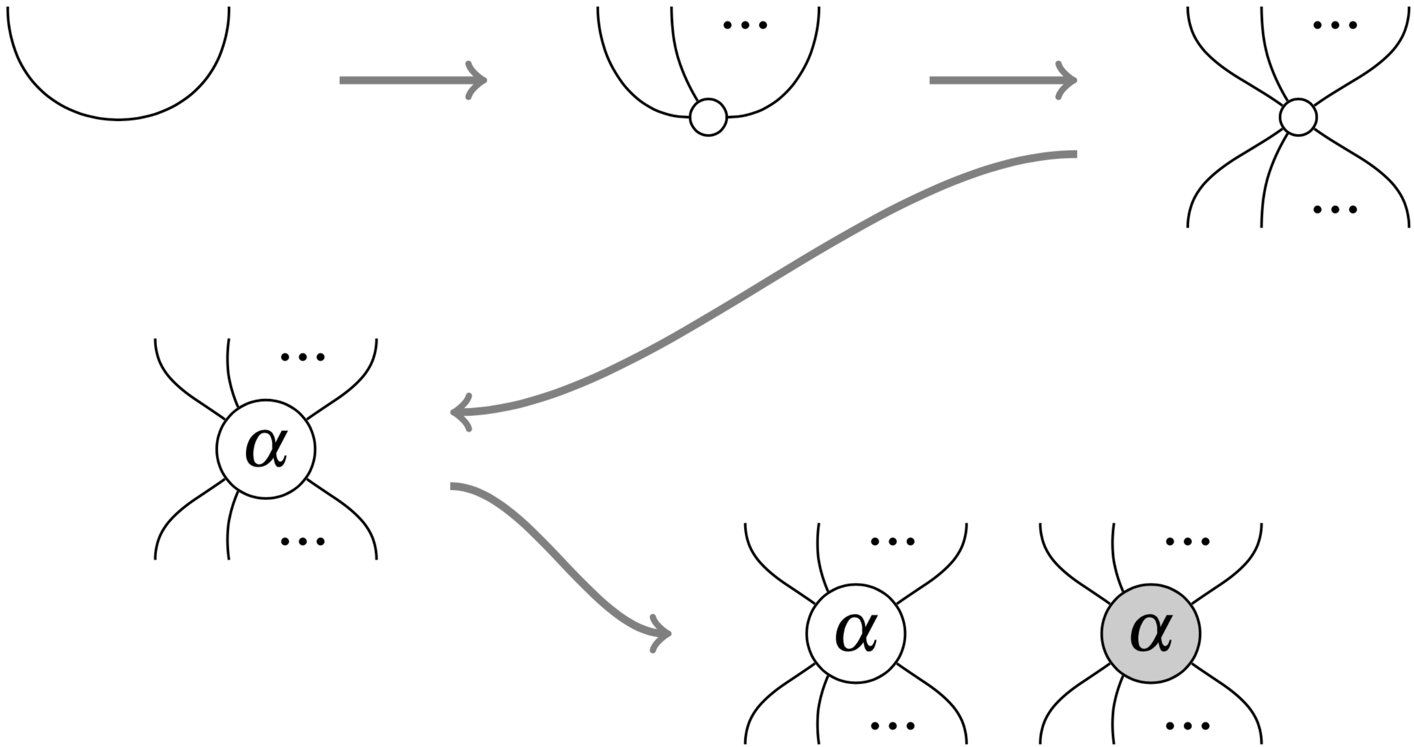
— structural evolution  —



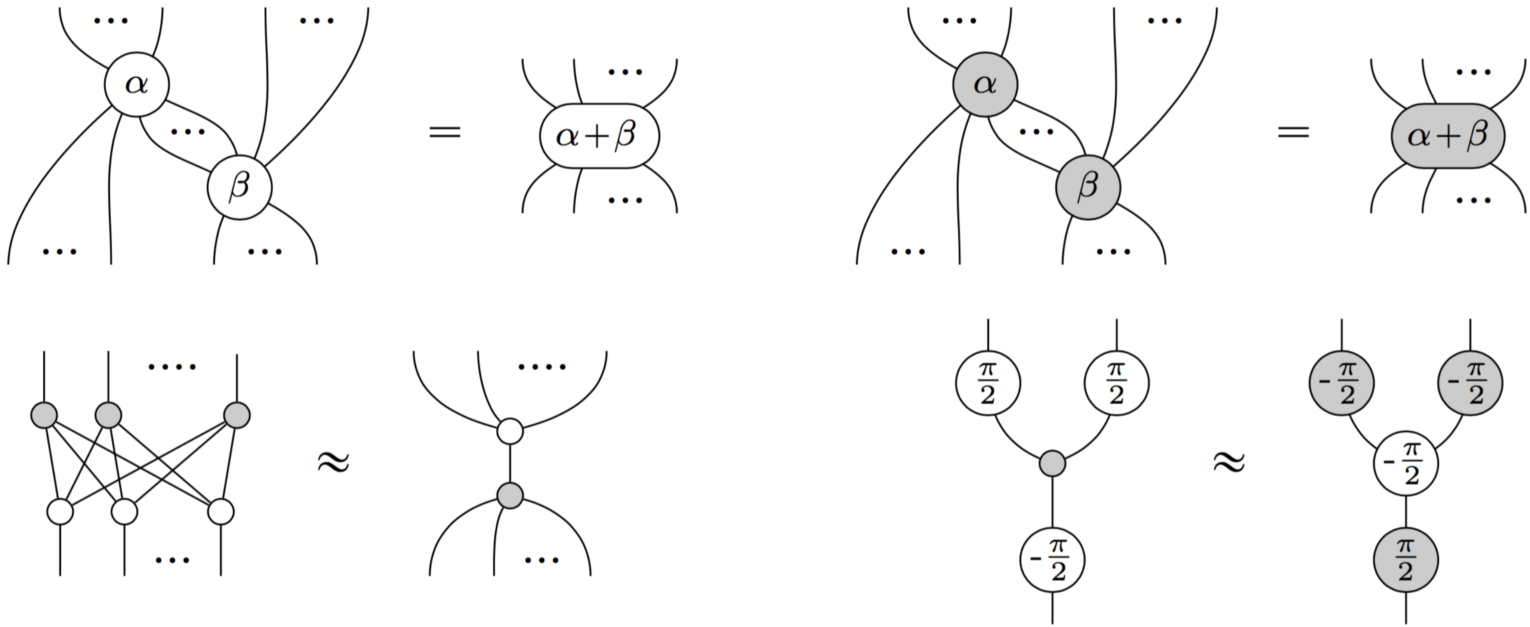
structural evolution 



— structural evolution —



– ZX-calculus –



B. Coecke & R. Duncan (2008) *Interacting quantum observables*. ICALP & NJP. arXiv:0906.4725

– *completeness* –

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M. Backens (2012) Any equational statement is provable in the stabiliser restriction of **ZX-calculus** if and only if it is provable for Hilbert spaces and linear maps.

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Kang Feng Ng and Quanlong Wang (2017) ... everything$^+$, even better...

E. Jeandel, S. Perdrix & R. Vilmart (51 minutes ago) ... everything, even^2 better...

Kang Feng Ng and Quanlong Wang (37 minutes ago) ... everything$^+$, even^3 better...

E. Jeandel, S. Perdrix & R. Vilmart (13.7 minutes ago) ... everything, even^4 better...

Kang Feng Ng and Quanlong Wang (3.4 seconds ago) ... everything$^+$, even^5 better...

Ongoing collaboration with:

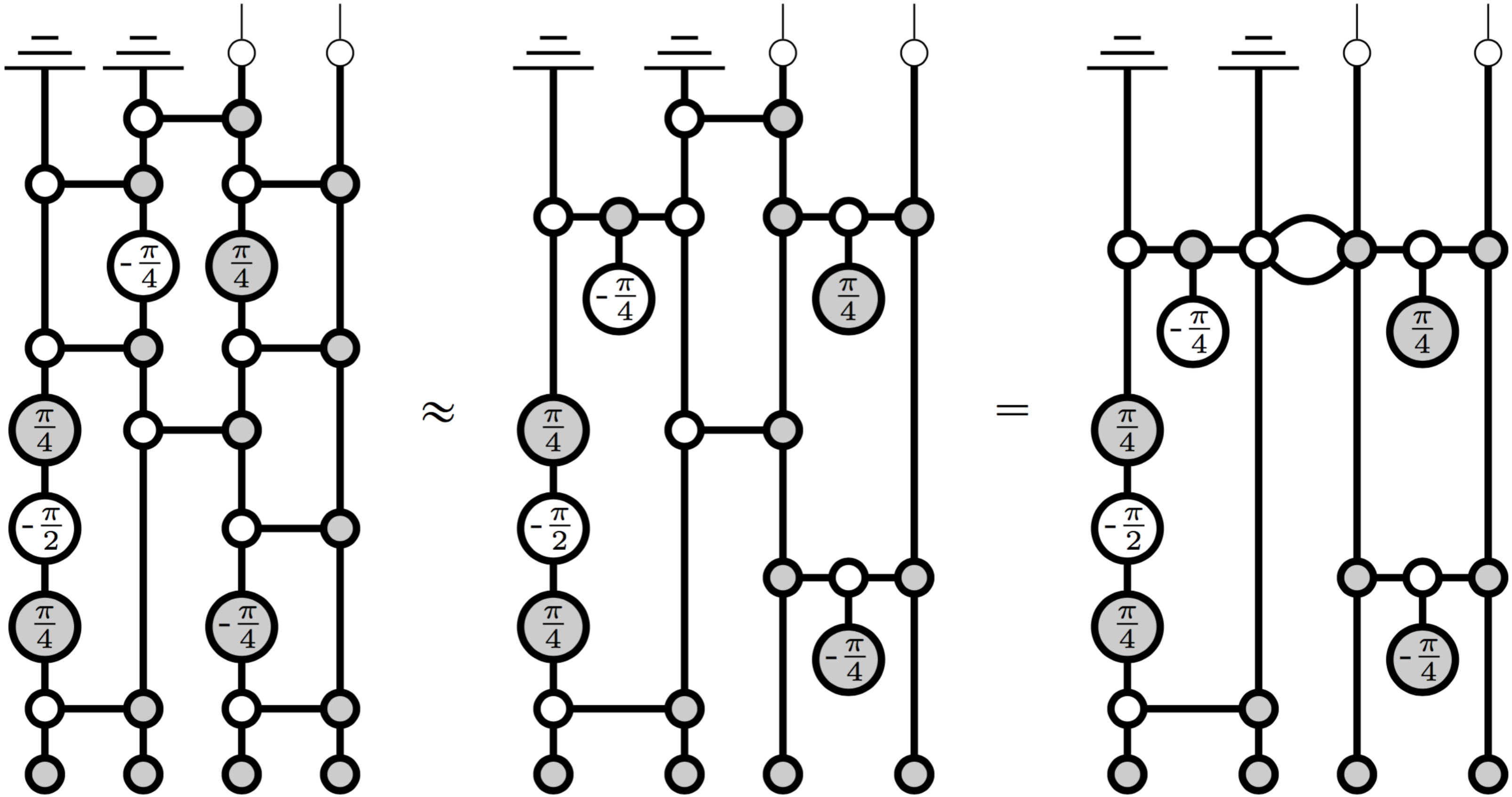
- **Cambridge Quantum Computing Inc.**

towards:

- **architecture-independent**
- **exact-efficient**

quantum compiler.

circuit rewriting :=



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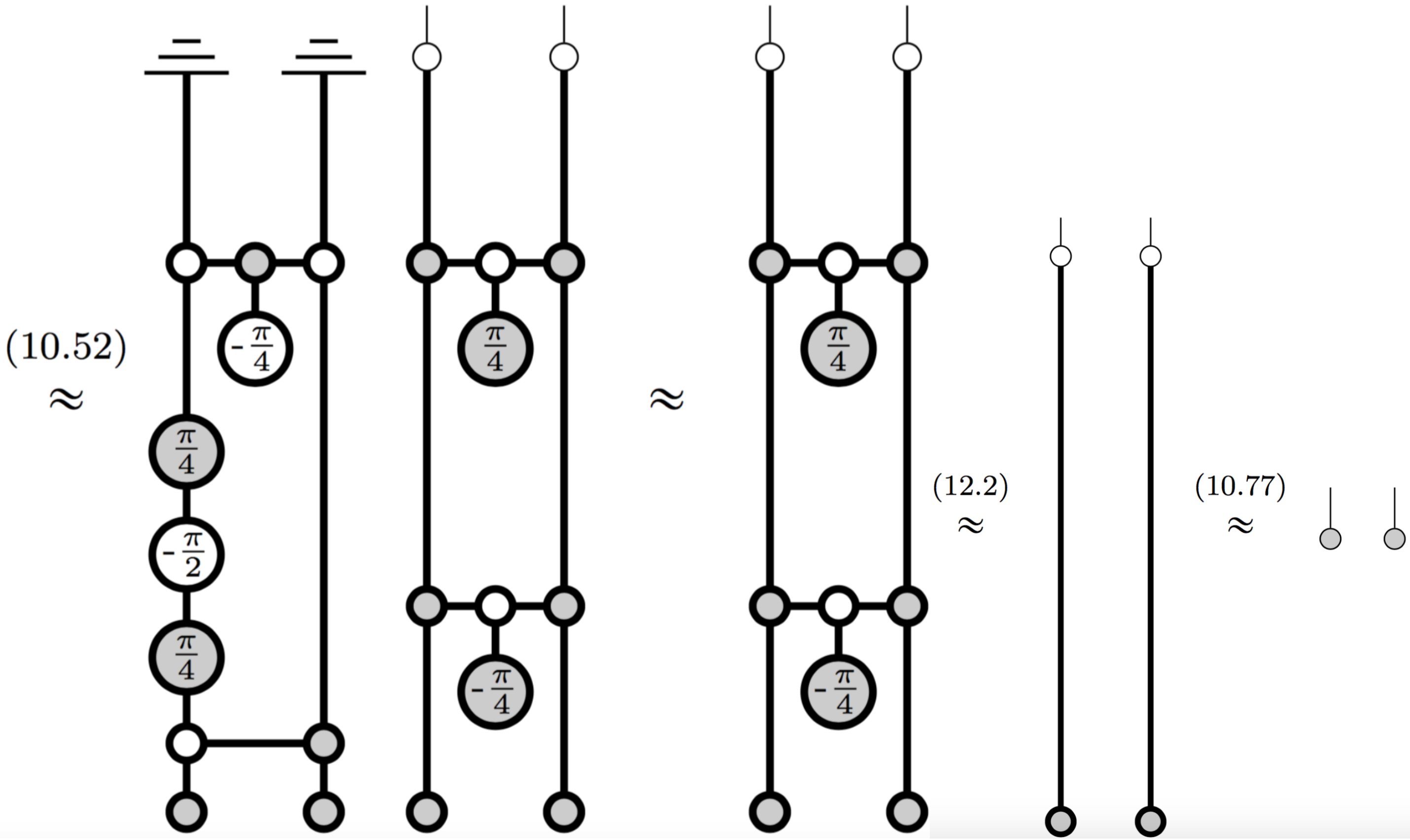
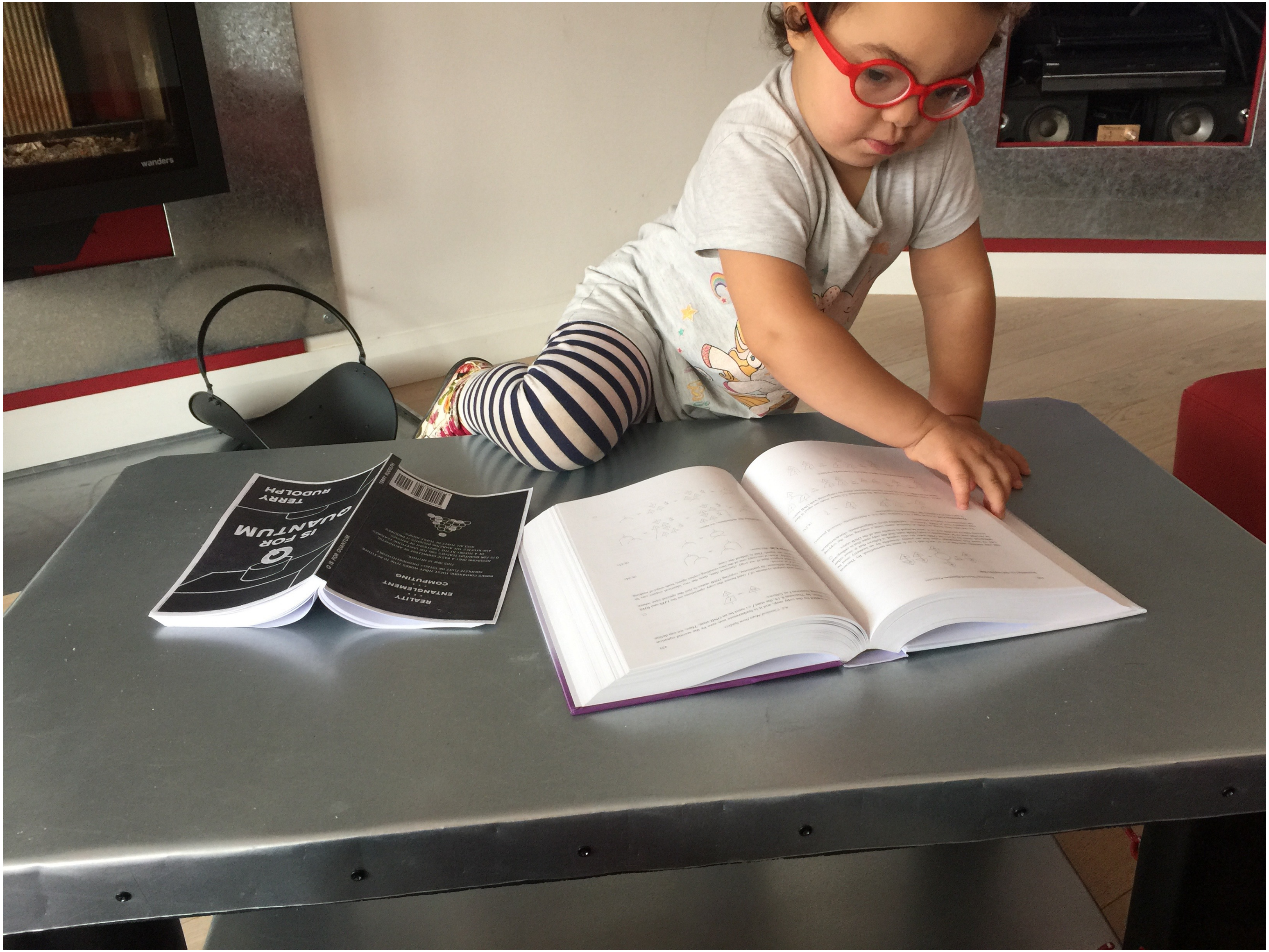
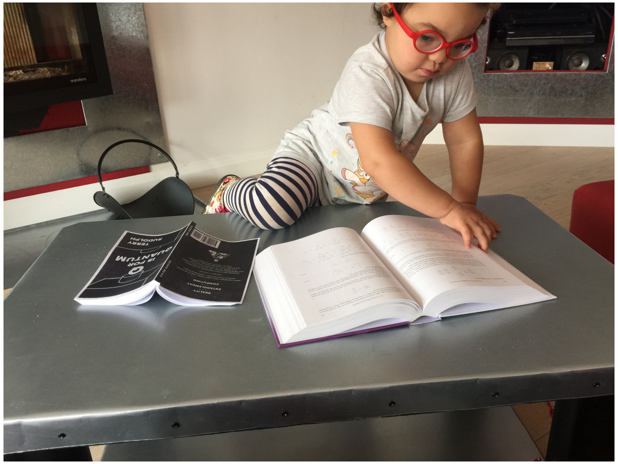


Any age restrictions?



EXPERIMENTS THIS SUMMER !



KIDS OUTPERFORM OXFORD STUDENTS AND DISCOVER QUANTUM FEATURES THAT TOOK TOP SCIENTISTS 60y